

# The elementary non-abelian self-dual string and the (2,0)-theory

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INFN Torino, 9.5.2017

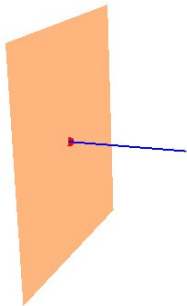
Based on:

- CS & L Schmidt, [arXiv:1705.02353](https://arxiv.org/abs/1705.02353) (today)

# Motivation: The Dynamics of Multiple M5-Branes

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To understand M-theory, an effective description of M5-branes would be very useful.

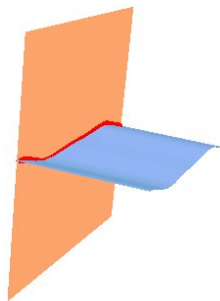


## D-branes

- D-branes **interact** via strings.
- Effective description: theory of **endpoints**
- Parallel transport of these: **Gauge theory**
- Study string theory **via gauge theory**

## M5-branes

- M5-branes **interact** via M2-branes.
- Eff. description: theory of **self-dual strings**
- Parallel transport: **Higher gauge theory**
- Long sought **(2,0)**-theory a **HGT**?



# What we know about the $(2,0)$ -theory

Multiple M5-branes are described by a  $\mathcal{N} = (2,0)$  superconformal field theory.

What we know about 6d  $\mathcal{N} = (2,0)$  SCFT:

- String theory considerations: **conformal fixed point in 6d**  
Witten, Strominger 1995
- Field content:  $\mathcal{N} = (2,0)$  **supermultiplet** in 6d:
  - a **self-dual 3-form field strength**
  - five (Goldstone) **scalars**
  - **fermionic partners**
- A theory of essentially **tensionless light strings**
- Supergravity **decouples**, so study string dynamics separately
- Observables: **Wilson surfaces**, i.e. parallel transport of strings
- **No Lagrangian description** known
- As important as  $\mathcal{N} = 4$  **super Yang-Mills** for string theory
- Huge interest in string theory: **AGT, AdS<sub>7</sub>-CFT<sub>6</sub>, S-duality, ...**
- Mathematics: **Geom. Langlands, Khovanov Homology, ...**

Parallel transport of particles in representation of gauge group  $G$ :

- holonomy functor  $\text{hol} : \text{path } \gamma \mapsto \text{hol}(\gamma) \in G$
- $\text{hol}(\gamma) = P \exp(\int_{\gamma} A)$ ,  $P$ : path ordering, trivial for  $U(1)$ .

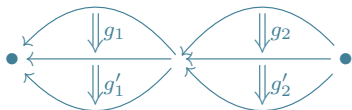
Parallel transport of strings with gauge group  $U(1)$ :

- map  $\text{hol} : \text{surface } \sigma \mapsto \text{hol}(\sigma) \in U(1)$
- $\text{hol}(\sigma) = \exp(\int_{\sigma} B)$ ,  $B$ : connective structure on gerbe.

Nonabelian case:

- much more involved!
- no straightforward definition of surface ordering

Imagine **parallel transport** of string with gauge degrees in  $\text{Lie}(\mathbf{G})$ :



Consistency of parallel transport requires:

$$(g'_1 g'_2)(g_1 g_2) = (g'_1 g_1)(g'_2 g_2)$$

This renders group  $\mathbf{G}$  **abelian**.

Eckmann and Hilton, 1962  
Physicists 80'ies and 90'ies

Way out: **2-categories**, **Higher Gauge Theory**.

Two operations  $\circ$  and  $\otimes$  satisfying **Interchange Law**:

$$(g'_1 \otimes g'_2) \circ (g_1 \otimes g_2) = (g'_1 \circ g_1) \otimes (g'_2 \circ g_2) .$$

Standard **objection** beyond the previous no-go theorem:

- theory at conformal fixed points  $\Rightarrow$  **no dimensionful parameter**
- fixed points are isolated  $\Rightarrow$  **no dimensionless parameter**
- **“No parameters  $\Rightarrow$  no classical limit  $\Rightarrow$  no Lagrangian.”**

Answers:

- Same arguments for **M2-brane** Schwarz, 2004
- There, integer parameters arose from **orbifold**  $\mathbb{R}^8/\mathbb{Z}_k$
- **Same should happen for M5-branes**
- Even if no Lagrangian, **BPS-states** may exist classically  
 $\Rightarrow$  **“self-dual strings”**
- Even if not, study **quantum features** of related theories.

## Focus on Self-Dual Strings, BPS states in (2,0)-theory.

Observations:

- Lift of D-brane interpretation of **BPS monopoles** to M-theory
- Involves “**categorified**” of “**higher**” version of gauge theory

Additional reasons for studying self-dual strings:

- Categorified **Integrability**
  - Twistor descriptions developed **CS, Martin Wolf 2012-2016**
  - Categorified Nahm Transform  $\Rightarrow$  **Categorified Dirac operator**
- Involves a **higher quantization** of  $S^3$ 
  - Important for **non-geometric backgrounds** in string theory
- Examples of categorified/**higher principal bundles**
  - Important for **mathematical progress**

## To formulate M5-brane theory: Need category theory

Some quotes:

- “We will need to use some very simple notions of category theory, an **esoteric subject** noted for its **difficulty** and **irrelevance**.”

G. Moore and N. Seiberg, 1989

- “We’ll only use as much category theory as is necessary.  
**Famous last words...**”

Roman Abramovich

- “Category theory is the subject where you can leave the **definitions as exercises**.”

John Baez



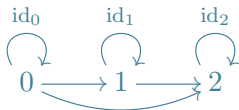
## Category theory:

- **Idea**: put mathematical objects on **same footing as maps**
- We do this in physics: **Gauge theory**:
  - Best strategy: **do not mod out gauge equivalence**
  - Work **in parallel** with gauge fields and transformations

## Categorification:

- Roughly: Replace sets by **categories**
- Leads to “**categories of maps/morphisms**”
- ... and **gauge transformations between** gauge transformations
- Known from physics: **BRST quantization**
  - **ghosts for ghosts**
  - BRST complex is an  $L_\infty$ -algebroid

Example:



Category  $\mathcal{C}$ :

- Classes of **Objects**  $\mathcal{C}_0 = \{0, 1, 2\}$  and **Arrows**  $\mathcal{C}_1$ :  $\mathcal{C}_1 \rightrightarrows \mathcal{C}_0$

- Associative composition** always possible:  $a \begin{array}{c} \xleftarrow{f} \\ \xleftarrow{h=f \circ g} \\ \xleftarrow{g} \end{array} b \leftarrow c$

- Each object has **identity morphism**:  $\begin{array}{c} \text{id}_a \\ \curvearrowright \\ a \end{array}, \quad \begin{array}{l} f \circ \text{id}_b = f \\ \text{id}_b \circ g = g \end{array}$

Examples:

- set**  $S$ :  $S \rightrightarrows S$
- Set**:  $\text{Set}_0$ : sets,  $\text{Set}_1$ : functions between sets
- Vect**:  $\text{Vect}_0$ : vector spaces,  $\text{Vect}_1$ : linear maps between them
- Mfd** $^\infty$ :  $\text{Mfd}_0^\infty$ : smooth manifolds,  $\text{Mfd}_1^\infty$ : smooth maps

## Internalization in a category $\mathcal{C}$

- Math. notion as **objects**, **arrows**, and **commutative diagrams**.
- **Restrict** objects/arrows to those of category  $\mathcal{C}$ .

Group  $G$  as morphisms:

$$\mathbb{1} : * \longrightarrow G, \quad (-)^{-1} : G \longrightarrow G, \quad m : G \times G \longrightarrow G$$

such that identities hold:

$$\begin{array}{ccc}
 G & \xrightarrow{(\mathbb{1}, \text{id})} & G \times G \\
 \text{id} \downarrow & \searrow \text{id} & \downarrow m \\
 G \times G & \xrightarrow{m} & G
 \end{array}
 , \text{ etc.}$$

Examples:

- Group (object) internal to **Top**: **topological group**
- Group (object) internal to **Mfd<sup>∞</sup>**: **Lie group**
- Group (object) internal to **Cat**: **strict 2-group**
- Category internal to **Mfd<sup>∞</sup>**: **2-space**
- Category internal to **Vect**: **2-vector space**

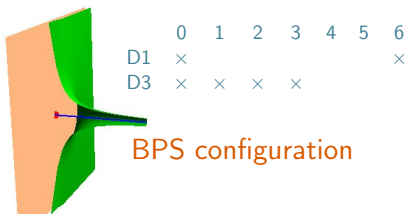
Categorification **not unique**, but can categorify almost anything:

- Categorified groups or **2-groups**
- Categorified Lie algebras or **Lie 2-algebras**
- Categorified principal bundles or **principal 2-bundles**
- Categorified **connections**

Issues:

- **Very complicated structures** packaged in very compact form
- Unpacking of definitions is **complicated**, requires **much work**
- Lots and lots of **equivalences/isomorphisms**
- Usually: **Few available examples**

**self-dual strings (BPS in (2,0)-theory): important examples.**



Perspective of D3:

Bogomolny monopole eqn.

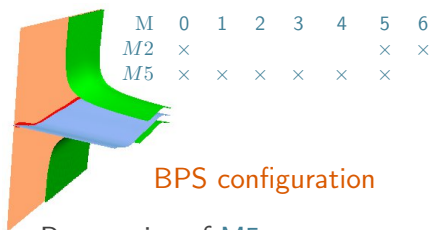
$$F = \nabla^2 = *\nabla\Phi \text{ on } \mathbb{R}^3$$

↕ Nahm transform ↕

Perspective of D1:

Nahm eqn.

$$\frac{d}{dx^6} X^i + \varepsilon^{ijk} [X^j, X^k] = 0$$



Perspective of M5:

Abelian Self-dual string eqn.

$$H := dB = *d\Phi \text{ on } \mathbb{R}^4$$

↕ genlzd. Nahm transform (?) ↕

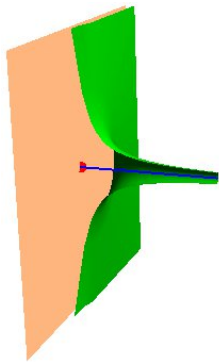
Perspective of M2:

Hoppe-Basu-Harvey eqn. (??)

$$\frac{d}{dx^6} X^\mu + \varepsilon^{\mu\nu\rho\sigma} [X^\nu, X^\rho, X^\sigma] = 0$$

# Side remark: Fuzzy Spheres from Monopoles

dim	0	1	2	3	...	6
D1	×					×
D3	×	×	×	×		



$N$  D1-branes ending on D3-branes:

A Monopole appears.

$X^i \in \mathfrak{u}(N)$ : transverse fluctuations

Nahm equation:

$$\frac{d}{dx^6} X^i + \varepsilon^{ijk} [X^j, X^k] = 0$$

Solution:  $X^i = r(x^6) G^i$  with

$$r(s) = \frac{1}{x^6}, \quad G^i = \varepsilon^{ijk} [G^j, G^k]$$

Nahm, Diaconescu, Tsimpis

Analogously, M2-M5-brane configuration should yield a fuzzy  $S^3$ .

Recall:

- Abelian Dirac Monopole: singular on  $\mathbb{R}^3$
- Non-abelian 't Hooft–Polyakov Monopole: non-singular on  $\mathbb{R}^3$
- Abelian Dirac Monopole: can add solutions (non-interacting)
- Abelian Self-Dual String: singular on  $\mathbb{R}^4$
- Abelian Self-Dual String: can add solutions (non-interacting)

Goal: Non-abelian self-dual string with

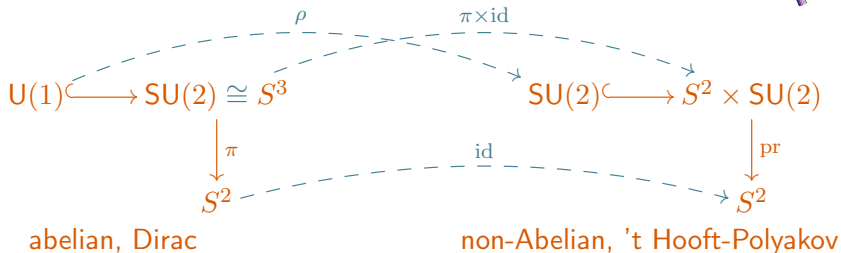
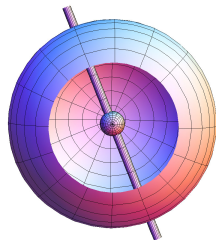
- non-singular solution on  $\mathbb{R}^4$
- interacting solution

Steps:

- Identify gauge structure
- Identify equations of motion
- Find at least elementary (charge 1) solution

## Monopoles

- Solution to **Bogomolny equation**  $F = *\nabla\phi$
- Abelian: singular on  $\mathbb{R}^3$ , **Dirac strings**
- Principal bundle over  $S^2$
- Non-Abelian: non-singular on  $\mathbb{R}^3$



$\Rightarrow$  Choose **SU(2)**, as trivialization possible.



# Lifting Monopoles to Self-Dual Strings

D-brane interpretation of **monopole in type IIB**:

<b>IIB</b>	0	1	2	3	4	5	6
<i>D1</i>	×						×
<i>D3</i>	×	×	×	×			

**T-dualize** along  $x^5$ :

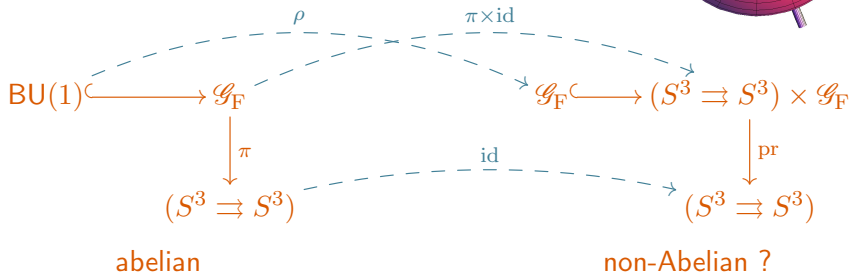
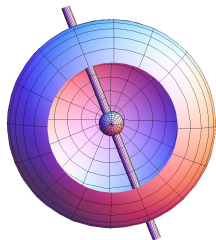
<b>IIA</b>	0	1	2	3	4	5	6
<i>D2</i>	×					×	×
<i>D4</i>	×	×	×	×		×	

Interpret  $x^4$  as **M-theory** direction:

<b>M</b>	0	1	2	3	4	5	6
<i>M2</i>	×					×	×
<i>M5</i>	×	×	×	×	×	×	

## Self-Dual Strings

- Abelian: singular on  $\mathbb{R}^4$ , Dirac strings
- Solution to  $H = *\nabla\phi$
- Gerbe over  $S^3$
- Non-Abelian: ?

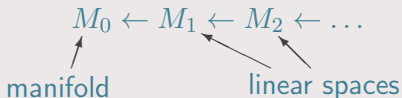


$\Rightarrow$  Choose  $\mathcal{G}_F$ , with 2-group structure: **String 2-group**  
 (many other reasons for this)

- Monopole/instanton solutions: gauge group from **spin group**  
 $\text{Spin}(3) \cong \text{SU}(2)$ ,  $\text{Spin}(4) \cong \text{SU}(2) \times \text{SU}(2)$
- **Higher analogue** of the spin group: **String group**  
Stolz, Teichner, Witten, ...
- Def. via **Whitehead tower** (iteratively delete homotopy groups)  
 $\dots \rightarrow \text{String}(n) \rightarrow \text{Spin}(n) \rightarrow \text{Spin}(n) \rightarrow \text{SO}(n) \rightarrow \text{O}(n)$
- Definition only **up to homotopy**, as a group:  $\infty$ -dimensional
- 2-group models:
  - $\infty$ -dimensional strict 2-group      Baez et al., Nikolaus et al.
  - finite-dimensional quasi 2-group      Schommer-Pries
- Higher gauge theory developed      Demessie, CS (2016)
- Many reasons: Gauge 2-group for M5-branes is  $\text{String}(E_8)$       Anahori, Stolz (2004)

## N-manifolds, NQ-manifold

- **$\mathbb{N}_0$ -graded manifold** with coordinates of degree  $0, 1, 2, \dots$



- **NQ-manifold**: vector field  $Q$  of degree 1,  $Q^2 = 0$
- **Physicists**: think ghost numbers, BRST charge, SFT

## Examples:

- **Tangent algebroid**  $T[1]M$ ,  $\mathcal{C}^\infty(T[1]M) \cong \Omega^\bullet(M)$ ,  $Q = d$
- **Lie algebra**  $\mathfrak{g}[1]$ , coordinates  $\xi^a$  of degree 1:

$$Q = -\frac{1}{2} f_{ab}^c \xi^a \xi^b \frac{\partial}{\partial \xi^c} \quad , \quad \text{Jacobi identity} \Leftrightarrow Q^2 = 0$$

Lie  $n$ -algebroid or  $n$ -term  $L_\infty$ -algebroid:

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \dots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \dots$$

Lie  $n$ -algebra or  $n$ -term  $L_\infty$ -algebra:

$$* \leftarrow M_1 \leftarrow M_2 \leftarrow \dots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \dots$$

Important example: Lie 2-algebra

- Graded vector space:  $W[1] \leftarrow V[2]$
- Coordinates:  $w^a$  of degree 1 on  $W[1]$ ,  $v^i$  of degree 2 on  $V[2]$
- Most general vector field  $Q$  of degree 1:

$$Q = -m_i^a v^i \frac{\partial}{\partial w^a} - \frac{1}{2} m_{ab}^c w^a w^b \frac{\partial}{\partial w^c} - m_{ai}^j w^a v^i \frac{\partial}{\partial v^j} - \frac{1}{3!} m_{abc}^i w^a w^b w^c \frac{\partial}{\partial v^i}$$

- Induces “brackets”/“higher products”:

$$\mu_1(\tau_i) = m_i^a \tau_a, \quad \mu_2(\tau_a, \tau_b) = m_{ab}^c \tau_c, \quad \dots, \quad \mu_3(\tau_a, \tau_b, \tau_c) = m_{abc}^i \tau_i$$

- $Q^2 = 0 \Leftrightarrow$  Homotopy Jacobi identities, e.g.  $\mu_1(\mu_1(-)) = 0$
- Failure of Jacobi identity:  $\mu_2(x, \mu_2(y, z)) + \dots = \mu_1(\mu_3(x, y, z))$

- Recall:  $\mathcal{G}_F$  can be extended to String 2-group model
- Lie differentiate (e.g. Demessie, CS (2016))
- Result: String Lie 2-algebra  $\mathbf{string}(3) = \mathbb{R}[1] \rightarrow \mathfrak{su}(2)$  with

$$Q\xi^\alpha = -\frac{1}{2}f_{\beta\gamma}^\alpha \xi^\beta \xi^\gamma, \quad Qb = -\frac{1}{3!}f_{\alpha\beta\gamma} \xi^\alpha \xi^\beta \xi^\gamma.$$

or

$$\mu_2(x_1, x_2) = [x_1, x_2], \quad \mu_3(x_1, x_2, x_3) = (x_1, [x_2, x_3])$$

where  $x_{1,2,3} \in \mathfrak{su}(2)$ .

- Ideas: Atiyah, Strobl et al., Sati, Schreiber, Stasheff
- Recall: Chevalley-Eilenberg algebra of Lie algebra  $\mathfrak{g}$ :

$$\text{CE}(\mathfrak{g}) = \mathcal{C}^\infty(\mathfrak{g}[1]) , \quad Q\xi^\alpha = -\frac{1}{2}f_{\beta\gamma}^\alpha \xi^\beta \xi^\gamma$$

- Double to Weil algebra (a.k.a. as Free Differential Algebra)  
 $W(\mathfrak{g}) := \mathcal{C}^\infty(T[1]\mathfrak{g}[1]) , \quad Q = Q_{\text{CE}} + \sigma , \quad \sigma Q_{\text{CE}} = -Q_{\text{CE}}\sigma$
- Potentials/curvatures/Bianchi identities from dga-morphisms

$$(A, F) : W(\mathfrak{g}) \rightarrow \Omega^\bullet(M) = W(M)$$

$$\xi^\alpha \mapsto A^\alpha$$

$$(\sigma\xi^\alpha) = Q\xi^\alpha + \frac{1}{2}f_{\beta\gamma}^\alpha \xi^\beta \xi^\gamma \mapsto F^\alpha = (dA + \frac{1}{2}[A, A])^\alpha$$

$$Q(\sigma\xi^\alpha) = -f_{\beta\gamma}^\alpha (\sigma\xi^\alpha) \xi^\beta \mapsto (\nabla F)^\alpha = 0$$

- Gauge transformations: homotopies between dga-morphisms
- Topological invariants: invariant polynomials in  $W(\mathfrak{g})$

- Recall: **Chevalley-Eilenberg algebra** of String Lie 2-algebra  $\mathfrak{g}$ :

$$\mathrm{CE}(\mathfrak{g}) = \mathcal{C}^\infty(\mathbb{R}[2] \rightarrow \mathfrak{su}(2)[1]) ,$$

$$Q\xi^\alpha = -\frac{1}{2}f_{\beta\gamma}^\alpha \xi^\beta \xi^\gamma \quad \text{and} \quad Qb = \frac{1}{3!}f_{\alpha\beta\gamma} \xi^\alpha \xi^\beta \xi^\gamma .$$

- Double to **Weil algebra**

$$W(\mathfrak{g}) := \mathcal{C}^\infty(T[1]\mathfrak{g}[1]) , \quad Q = Q_{\mathrm{CE}} + \sigma , \quad \sigma Q_{\mathrm{CE}} = -Q_{\mathrm{CE}}\sigma$$

- Potentials/curvatures/Bianchi identities** from **dga-morphisms**

$$(A, B, F, H) : W(\mathfrak{g}) \rightarrow \Omega^\bullet(M) = W(M)$$

$$\xi^\alpha \mapsto A^\alpha \in \Omega^1(M) \quad \text{and} \quad b \mapsto B \in \Omega^2(M)$$

$$(\sigma\xi^\alpha) = Q\xi^\alpha + \frac{1}{2}f_{\beta\gamma}^\alpha \xi^\beta \xi^\gamma \mapsto F^\alpha = (dA + \frac{1}{2}[A, A])^\alpha$$

$$(\sigma b) = Qb - \frac{1}{3!}f_{\alpha\beta\gamma} \xi^\alpha \xi^\beta \xi^\gamma \mapsto H = dB - \frac{1}{3!}(A, [A, A])$$

- Bianchi identities:**  $\nabla F = 0$  and  $dH = -\frac{1}{2}(dA, [A, A])$
- Gauge trafos** and **Top. invariants** derived as above



Higher Gauge potential for `string(3)`: CS, Schmidt (2017)

$$A \in \Omega^1(\mathbb{R}^4) \otimes \mathfrak{su}(2) , \quad B \in \Omega^2(\mathbb{R}^4) \otimes \mathbb{R} ,$$

Add Higgs field:

$$\phi \in \Omega^0(\mathbb{R}^4) \otimes \mathbb{R}$$

Equations of motion:

- Straightforward discussion from dga-morphisms yields:

$$H = dB - \frac{1}{3!}(A, [A, A]) = *\nabla\phi , \quad F = dA + \frac{1}{2}[A, A] = 0 .$$

- **Problem 0:** Only abelian solutions:  $\Rightarrow$  Drop  $F = 0$
- **Problem 1:** Gauge invariance unclear
- **Problem 2:** “Bianchi identity:” vs Anomaly cancellation

$$dH = -\frac{1}{3!}d(A, [A, A]) = *\square\phi \quad \text{vs} \quad dH = F \wedge F$$

- Need to **modify equations** of motion

**Field Content** with values in  $\mathfrak{string}(3)$ :

$$A \in \Omega^1(\mathbb{R}^4) \otimes \mathfrak{su}(2), \quad B \in \Omega^2(\mathbb{R}^4) \otimes \mathbb{R}, \quad \phi \in \Omega^0(\mathbb{R}^4) \otimes \mathbb{R}$$

Use categorical equivalences:

- $SU(N)$ -bundle:  $U(N)$ -bundle + trivialization of  $U(1)$ -factor
- $\mathfrak{String}(3)$ -bundle as  $\mathfrak{Spin}(3)$ -bundle +  $\mathfrak{BBU}(1)$ -factor  
Sati, Schreiber, Stasheff (2009)

- Upshot: add  $(A, F)$  to  $H$ , which yields:

$$H = dB + \frac{1}{2}(A, dA) + \frac{1}{3!}(A, [A, A]) = *d\phi \Rightarrow dH = (F, F) = *\square\phi$$

- $\phi$  should “know” all about configuration, thus demand also

$$F = dA + \frac{1}{2}[A, A] = *F$$

- Check 1: **Nice reduction** of 2nd equation to monopoles on  $\mathbb{R}^3$
- First equation requires little more work, **also reduces perfectly**
- Check 2: **BPS equations** for (1,0)-model (more later)

**Bogomolny's trick** familiar from instanton/monopoles works, too:

$$S = \int_{\mathbb{R}^4} H \wedge *H + d\varphi \wedge *d\varphi + (F, *F)$$

Rewrite:

$$S = \int_{\mathbb{R}^4} (H - *d\varphi) \wedge *(H - *d\varphi) + 2H \wedge d\varphi + \frac{1}{2}((F - *F), *(F - *F)) + (F, F)$$

Minimum/Bogomolny bound:

$$H = *d\varphi, \quad F = *F$$

Topological invariants

$$S = 2 \int_{\mathbb{R}^4} H \wedge d\varphi + \int_{\mathbb{R}^4} (F, F) = \int_{S_\infty^3} H + \int_{\mathbb{R}^4} (F, F)$$

Equations of motion:

$$F = dA + \frac{1}{2}[A, A] = *F \quad \text{and} \quad H = dB + CS(A) = *d\phi$$

Solution:

$$A_\mu(x) = \frac{1}{i} \frac{\eta_{\mu\nu}^i \sigma_i (x - x_0)^\nu}{\rho^2 + (x - x_0)^2}, \quad B = 0, \quad \varphi = \frac{((x - x_0)^2 - 2\rho^2)}{((x - x_0)^2 + \rho^2)^2}$$

cf. also [Akyol, Papadopoulos 2012](#)

Consistency checks:

- Globally **non-singular** on  $\mathbb{R}^4$
- **Approaches abelian solution**  $\frac{1}{x^2}$  as  $x \rightarrow \infty$
- Moduli: same as instanton:
  - **Position**
  - **Size** (conformal symmetry of  $F = *F$ )
  - Residual **global SU(2) gauge symmetry**

Alternative, but **equivalent form of String Lie 2-algebra**:

$$\text{string}_{\hat{\Omega}} = \left( \Omega\mathfrak{su}(2) \oplus \mathbb{R} \rightarrow P_0\mathfrak{su}(2) \right)$$

- $\Omega\mathfrak{su}(2)$  and  $P_0\mathfrak{su}(2)$ : based loop and path spaces
- **Quasi-Isomorphism** between both models
- Should be reflected in equivalence of **physics**

We find:

- **Fields**:  $A \in \Omega^1(\mathbb{R}^4) \otimes P_0\mathfrak{su}(2)$ ,  $B \in \Omega^2(\mathbb{R}^4) \otimes (\Omega\mathfrak{su}(2) \oplus \mathbb{R})$   
and Higgs field  $\varphi \in \Omega^0(\mathbb{R}^4) \otimes (\Omega\mathfrak{su}(2) \oplus \mathbb{R})$
- **Equations of motion**, modified for twisted string structures:

$$\mathcal{F} := dA + \frac{1}{2}\mu_2(A, A) + \mu_1(B)$$

$$H := dB + \mu_2(A, B) - \kappa(A, \mathcal{F}) = *\nabla\varphi$$

- Explicit 1:1-map between **gauge equivalence classes**
- $\Rightarrow$  **Physics respects categorical equivalence**

## BPS states work, what about classical (2,0)-theory?

Recall:

- 6d (1,0)-model derived from tensor hierarchies  
Samtleben, Sezgin, Wimmer (2011)
- Issue 1: Choice of gauge structure unclear
- Issue 2: cubic interactions
- Issue 3: scalar fields with wrong sign kinetic term
- Issue 4: Self-duality of 3-form imposed by hand

Previous observation:

- Gauge structure has underlying Lie 3-algebra + extra struct.  
Palmer, CS (2013), Samtleben et al. (2014)

New:

Schmidt, CS (2017)

- **Idea**: use  $\mathfrak{string}(3)$  as gauge structure in this model
- Issue: need suitable notion of **inner product** for action
- **Appropriate inner products** on  $L_\infty$ -algebras: **symplectic forms**
- Consequence: Double twisted  $\mathfrak{string}(3)$

$$\mathbb{R} \rightarrow \mathbb{R} \rightarrow \mathfrak{su}(2)$$

to something symplectic:

$$\mathbb{R} \oplus \mathfrak{su}(2)^* \rightarrow \mathbb{R} \oplus \mathbb{R}^* \rightarrow \mathfrak{su}(2) \oplus \mathbb{R}^*$$

- This carries **natural inner product**
- Can be extended to **Lie 3-algebra**
- Has necessary **extra structure**

Field content:

- **(1,0) tensor multiplet**  $(\phi, \chi^i, B)$ , values in  $\mathbb{R}^2$ ,  $\phi = \phi_s + \phi_r, \dots$
- **(1,0) vector multiplet**  $(A, \lambda^i, Y^{ij})$ , values in  $\mathfrak{su}(2) \oplus \mathbb{R}$
- **C-field**, values in  $\mathbb{R} \oplus \mathfrak{su}(2)^*$

Action (schematically):

$$S = \int_{\mathbb{R}^{1,5}} \left( \mathcal{H} \wedge * \mathcal{H} + d\phi \wedge * d\phi - * \langle \bar{\chi}, \not{\partial} \chi \rangle + \mathcal{H}_s \wedge * \langle \bar{\lambda}, \gamma_{(3)} \lambda \rangle + * \langle Y, \bar{\lambda} \rangle \chi_s \right. \\ \left. + \phi_s ((\mathcal{F}, * \mathcal{F}) - *(Y, Y) + *(\bar{\lambda}, \not{\nabla} \lambda)) + \langle \bar{\lambda}, \mathcal{F} \rangle \wedge * \gamma_{(2)} \chi_s \right. \\ \left. + \mu_1(C) \wedge \mathcal{H}_s + B_s \wedge (\mathcal{F}, \mathcal{F}) + B_s \wedge ([A, A], [A, A]) \right)$$

This solves problems 1 and 2:

- **Choice of gauge structure** for ADE-(2,0)-theories **clear**.
- **No cubic interaction term** for scalar fields



Crucial consistency check: **Reduction to D-branes/SYM theory**

$$S = \int_{\mathbb{R}^{1,5}} \left( \mathcal{H} \wedge * \mathcal{H} + d\phi \wedge * d\phi - * \langle \bar{\chi}, \not{\partial} \chi \rangle + \mathcal{H}_s \wedge * \langle \bar{\lambda}, \gamma_{(3)} \lambda \rangle + * \langle Y, \bar{\lambda} \rangle \chi_s \right. \\ \left. + \phi_s ((\mathcal{F}, * \mathcal{F}) - *(Y, Y) + *(\bar{\lambda}, \not{\nabla} \lambda)) + \langle \bar{\lambda}, \mathcal{F} \rangle \wedge * \gamma_{(2)} \chi_s \right. \\ \left. + \mu_1(C) \wedge \mathcal{H}_s + B_s \wedge (\mathcal{F}, \mathcal{F}) + B_s \wedge ([A, A], [A, A]) \right)$$

- Compactify along  $x^{10}$ ,  $x^9$  interpret as  $\langle \phi_s \rangle = \frac{1}{\pi^2 R_{10} R_9} = \frac{1}{\pi^2 R_{10}^2}$
- **Strong coupling expansion** around  $\langle \phi_s \rangle$  (cf. M2  $\rightarrow$  D2)
- Coupling constants:

$$\int_{T^2} \text{dvol}(T^2) \frac{1}{16\pi^2 R_{10}^2} = \frac{4\pi^2 R^9 R^{10}}{16\pi^2 R_{10}^2} = \frac{1}{4} \frac{R_9}{R_{10}} = \frac{1}{4g_s} = \frac{1}{4g_{\text{YM}}^2}$$

- This yields **4d  $\mathcal{N} = 2$  SYM** with gauge group **SU(2)**!

### Summary:

- Categorized structures appear naturally in M-theory
- Higher analogue of  $SU(2)$  is  $String(3)$
- There is a non-abelian self-dual string
- Better understanding of  $(1,0)$ -theory
- Dimensional reduction to  $\mathcal{N} = 4$  SYM works!

### Soon to come:

- ▷ Interpretation of fuzzy  $S^3$
- ▷ Categorized Integrability
- ▷ Access new superconformal field theories
- ▷ Better understanding of M-theory

# The elementary non-abelian self-dual string and the (2,0)-theory

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INFN Torino, 9.5.2017