

# Looking For the Classical (2,0)-Theory

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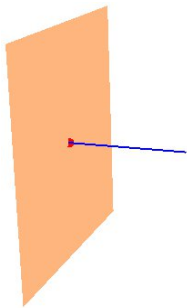
Based on work w. S Palmer, G Demessie, B Jurčo, M Wolf, P Ritter, L Schmidt:

- Higher Gauge Theory: [1203.5757](#), [1308.2622](#), [1311.1977](#), [1406.5342](#), [1512.07554](#), [1602.?????](#), [1602.?????](#)
- Integrability: [1105.3904](#), [1205.3108](#), [1305.4870](#), [1312.5644](#), [1403.7185](#)

# Motivation: The Dynamics of Multiple M5-Branes

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To understand M-theory, an effective description of M5-branes would be very useful.

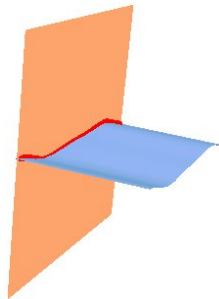


## D-branes

- D-branes **interact** via strings.
- Effective description: theory of **endpoints**
- Parallel transport of these: **Gauge theory**
- Study string theory **via gauge theory**

## M5-branes

- M5-branes **interact** via M2-branes.
- Eff. description: theory of **self-dual strings**
- Parallel transport: **Higher gauge theory**
- Can we study M-theory **via HGT?**



Multiple M5-branes are described by a  $\mathcal{N} = (2, 0)$  superconformal field theory.

What we know:

- String theory considerations: **conformal fixed point in 6d**  
Witten, Strominger 1995
- Field content:  $\mathcal{N} = (2, 0)$  **supermultiplet** in 6d:
  - a **self-dual 3-form field strength**
  - five (Goldstone) **scalars**
  - **fermionic partners**
- A theory of essentially **tensionless light strings**
- Supergravity **decouples**, so study string dynamics separately
- Observables: **Wilson surfaces**, i.e. parallel transport of strings
- **No Lagrangian description** known
- As important as  $\mathcal{N} = 4$  **super Yang-Mills** for string theory
- Huge interest in string theory: **AGT**, **AdS<sub>7</sub>-CFT<sub>6</sub>**, **S-duality**, ...
- Mathematics: **Geom. Langlands**, **Khovanov Homology**, ...

# Parallel Transport of Strings is Problematic

The lack of surface ordering renders a parallel transport of strings problematic.

Parallel transport of particles in representation of gauge group  $G$ :

- holonomy functor  $\text{hol} : \text{path } \gamma \mapsto \text{hol}(\gamma) \in G$
- $\text{hol}(\gamma) = P \exp(\int_{\gamma} A)$ ,  $P$ : path ordering, trivial for  $U(1)$ .

Parallel transport of strings with gauge group  $U(1)$ :

- map  $\text{hol} : \text{surface } \sigma \mapsto \text{hol}(\sigma) \in U(1)$
- $\text{hol}(\sigma) = \exp(\int_{\sigma} B)$ ,  $B$ : connective structure on gerbe.

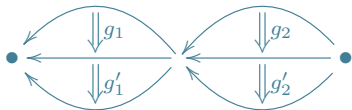
Nonabelian case:

- much more involved!
- no straightforward definition of surface ordering

# Naïve no-go theorem

Naively, there is no non-abelian parallel transport of strings.

Imagine **parallel transport** of string with gauge degrees in  $\text{Lie}(\mathbf{G})$ :



Consistency of parallel transport requires:

$$(g'_1 g'_2)(g_1 g_2) = (g'_1 g_1)(g'_2 g_2)$$

This renders group  $\mathbf{G}$  **abelian**.

Eckmann and Hilton, 1962  
Physicists 80'ies and 90'ies

Way out: **2-categories**, **Higher Gauge Theory**.

Two operations  $\circ$  and  $\otimes$  satisfying **Interchange Law**:

$$(g'_1 \otimes g'_2) \circ (g_1 \otimes g_2) = (g'_1 \circ g_1) \otimes (g'_2 \circ g_2) .$$

# Objection to a classical (2,0)-theory

Without coupling constant, there shouldn't be classical effective descriptions in M-theory.

Standard **objection** beyond the previous no-go theorem:

- theory at conformal fixed points  $\Rightarrow$  **no dimensionful parameter**
- fixed points are isolated  $\Rightarrow$  **no dimensionless parameter**
- **“No parameters  $\Rightarrow$  no classical limit  $\Rightarrow$  no Lagrangian.”**

Answers:

- Same arguments for **M2-brane** Schwarz, 2004
- There, integer parameters arose from **orbifold**  $\mathbb{R}^8/\mathbb{Z}_k$
- **Same should happen for M5-branes**
- Even if no Lagrangian, **BPS-states** may exist classically
- Even if not, study **quantum features** of related theories.

## 4-Slide Crash Course on Higher Gauge Theory

One easily constructs local higher gauge theory using  $NQ$ -manifolds.

## $N$ -manifolds, $NQ$ -manifold

- $\mathbb{N}$ -graded manifold with coordinates of degree  $0, 1, 2, \dots$

$$\begin{array}{c} M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \dots \\ \uparrow \qquad \qquad \qquad \swarrow \quad \searrow \\ \text{manifold} \qquad \qquad \qquad \text{linear spaces} \end{array}$$

- **Morphisms**  $\phi : M \rightarrow N$  are maps  $\phi^* : \mathcal{C}^\infty(N) \rightarrow \mathcal{C}^\infty(M)$
- **$NQ$ -manifold**: vector field  $Q$  of degree 1,  $Q^2 = 0$
- **Physicists**: think ghost numbers, BRST charge etc.

Examples:

- **Tangent algebroid**  $T[1]M$ ,  $\mathcal{C}^\infty(T[1]M) \cong \Omega^\bullet(M)$ ,  $Q = d$
- **Lie algebra**  $\mathfrak{g}[1]$ , coordinates  $\xi^a$  of degree 1:

$$Q = -\frac{1}{2} f_{ab}^c \xi^a \xi^b \frac{\partial}{\partial \xi^c}$$

Condition  $Q^2 = 0$  is equivalent to **Jacobi identity** for  $f_{ab}^c$



$NQ$ -manifolds provide an easy definition of  $L_\infty$ -algebras.

Lie  $n$ -algebroid or  $n$ -term  $L_\infty$ -algebroid:

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \dots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \dots$$

Lie  $n$ -algebra,  $n$ -term  $L_\infty$ -algebra or Lie  $n$ -algebra:

$$* \leftarrow M_1 \leftarrow M_2 \leftarrow \dots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \dots$$

Example: Lie 2-algebra

- $NQ$ -manifold:  $* \leftarrow W[1] \leftarrow V[2] \leftarrow * \leftarrow \dots$ , coords.  $w^a, v^i$
- Homological vector field:

$$Q = -m_i^a v^i \frac{\partial}{\partial w^a} - \frac{1}{2} m_{ab}^c w^a w^b \frac{\partial}{\partial w^c} - m_{ai}^j w^a v^i \frac{\partial}{\partial v^j} - \frac{1}{3!} m_{abc}^i w^a w^b w^c \frac{\partial}{\partial v^i}$$

- Structure constants: higher products  $\mu_i$  on  $W \leftarrow V[1]$

$$\mu_1(\tau_i) = m_i^a \tau_a, \quad \mu_2(\tau_a, \tau_b) = m_{ab}^c \tau_c, \quad \dots, \quad \mu_3(\tau_a, \tau_b, \tau_c) = m_{abc}^i \tau_i$$

- $Q^2 = 0$ : Higher or homotopy Jacobi identity, e.g.

$$\mu_2(w_1, \mu_2(w_2, w_3)) + \text{cycl.} = \mu_1(\mu_3(w_1, w_2, w_3))$$

One easily constructs local higher gauge theory using NQ-manifolds.

Ordinary gauge theory:

- **Gauge potential** from morphism of  $N$ -manifolds:

$$a : T[1]M \rightarrow \mathfrak{g}[1] \quad \longrightarrow \quad A_\mu^a dx^\mu := a^*(\xi^a)$$

- **Curvature**: failure of  $a$  to be morphism of NQ-manifold:

$$F^a := (d \circ a^* - a^* \circ Q)(\xi^a) = dA^a + \frac{1}{2} f_{bc}^a A^b \wedge A^c$$

- **Gauge transformations**: flat homotopies between morphisms.

Higher gauge theory:

- **Gauge potentials**  $T[1]M \rightarrow (W[1] \leftarrow V[2])$ :

$$A_\mu^a dx^\mu := a^*(w^a) \quad \text{and} \quad B_{\mu\nu}^i dx^\mu \wedge dx^\nu = a^*(v^i)$$

- **Curvature**: failure of  $a$  to be morphism of NQ-manifold:

$$\begin{aligned} \mathcal{F} &:= dA + \frac{1}{2} \mu_2(A, A) + \mu_1(B) \\ \mathcal{H} &:= dB + \mu_2(A, B) + \frac{1}{3!} \mu_3(A, A, A) \end{aligned}$$

# Local Higher Gauge Theories

The most interesting higher gauge theories for us live in 6 and 4 dimensions.

- “Fake curvature”:  $\mathcal{F} = dA + \frac{1}{2}\mu_2(A, A) - \mu_1(B) = 0$   
Vanishing makes parallel transport reparam. invariant.
- 3-form curvature:  $\mathcal{H} = dB + \mu_2(A, B) + \frac{1}{3!}\mu_3(A, A, A)$

## Gauge part of (2,0) theory

If (2,0) theory on  $\mathbb{R}^{1,5}$  is a higher gauge theory, then gauge part is:

$$\mathcal{H} = *\mathcal{H} , \quad \mathcal{F} = 0 .$$

## Non-Abelian Self-Dual Strings

BPS equation for (2,0) theory on  $\mathbb{R}^4$  ( $\sim$  monopoles in 4d SYM)

$$\mathcal{H} = *(d\Phi + \mu_2(A, \Phi)) , \quad \mathcal{F} = 0 .$$

## The Global Picture: Categorification

*We will need to use some very simple notions of category theory, an esoteric subject noted for its difficulty and irrelevance.*

G. Moore and N. Seiberg, 1989

*What does categorification mean?*

One of Jeff Harvey's questions to identify the "generation PhD>1999" at Strings 2013.

Categorification provides some guidelines in the construction of higher objects.

Category theory: excellent tool for deformations/generalizations.

Notions used: categorification, internalization and enrichment.

Idea: Mathematical objects are stuff, structures, structure eqns.

Translate as follows:

- stuff (sets) becomes categories
- structures (functions) become functors
- structure equations become structure isomorphisms

# Example: 2-Groups

Categorifying a group, we arrive at the notion of a 2-group.

## Group:

- **Stuff:** Underlying set  $G$ , unit  $\mathbb{1}$
- **Structure:** Multiplication, inverse
- **Structure equations:** associativity,  $g^{-1}g = \mathbb{1}$ ,  $\mathbb{1}g = g\mathbb{1} = g$

## 2-Group:

- **Stuff:** A category  $\mathcal{C}$ , unit object  $\mathbb{1}$
- **Structure:** Multiplication bifunctor  $\otimes$ , inverse functor  $\text{inv}$
- **Structure isomorphisms:**
  - $a_{x,y,z} : (x \otimes y) \otimes z \Rightarrow x \otimes (y \otimes z)$
  - $l_x : x \otimes \mathbb{1} \Rightarrow x$ ,  $r_x : \mathbb{1} \otimes x \Rightarrow x$
  - $\text{inv}(x) \otimes x \Rightarrow \mathbb{1} \leftarrow x \otimes \text{inv}(x)$

Example: **Strict 2-Group**  $G \times H \rightrightarrows G$ ,

- $a, l, r$  all trivial,  $\text{inv}(x) \otimes x = \mathbb{1} = x \otimes \text{inv}(x)$
- $\text{id}(g) = (g, \mathbb{1}_H)$ ,  $(g_1, h_1) \otimes (g_2, h_2) = (g_1 g_2, h_1(g_1 \triangleright h_2))$ , etc.

Descent data for principal bundles is encoded in a functor.

The cover  $\sqcup_a U_a$  of a manifold  $M$  encoded in the Čech groupoid:

$$\check{\mathcal{C}}(U) : \bigsqcup_{a,b} U_{ab} \rightrightarrows \bigsqcup_a U_a, \quad U_{ab} \circ U_{bc} = U_{ac}.$$

## Principal $G$ -bundle

Transition functions are nothing but a functor  $g : \check{\mathcal{C}}(U) \rightarrow (G \rightrightarrows *)$

$$\begin{array}{ccc} \bigsqcup U_{ab} & \xrightarrow{g_{ab}} & G \\ \Downarrow & & \Downarrow \\ \bigsqcup U_a & \xrightarrow{*} & * \end{array} \quad g_{ab}g_{bc} = g_{ac}$$

Equivalence relations: natural isomorphisms.

Use higher categories: Higher bundles including gerbes



Higher connections and finite gauge transformations are readily derived.

First option: **Integrate** infinitesimal description from above

- Procedure **rather involved**
- Result usually **only categorically equivalent** to expected one

Second option: **Differentiate gauge 2-group** Ševera

- Functors from manifolds to transition functions of principal  $G$ -bundles over  $M \times \mathbb{R}^{0|1} \rightarrow M$
- Moduli space is **N-manifold**  $\text{Lie}(G)[1]$
- Induced action of  $\text{Hom}(\mathbb{R}^{0|1}, \mathbb{R}^{0|1})$  on moduli space yields  $Q$ .
- Directly generalizes to  $L_\infty$ - and quasi-groupoids

Finite **gauge transformations**: B Jurčo, CS & M Wolf

- Natural transformations yield **Maurer-Cartan** forms etc.
- Readily read off **gauge transformations**

Context:

Many Suggested Models are Higher Gauge Theories

The ABJM model can be completed to a higher gauge theory.

- Most dualities in string theory between **Yang-Mills theories**.
- And in M-theory? **M2-branes**: Chern-Simons-matter theories  
**M5-branes**: Tensor-multiplet theories
- These can be put on **equal footing**. **S Palmer&CS, 1311.1997**

Step 1: The ABJM gauge structures / **hermitian 3-Lie algebras**

- are (strict) Lie 2-algebras. **S Palmer&CS, 1203.5757**
- **but**:  $\mu_1 = 0$ , thus  $F = \mu_1(B) = 0$ .
- Recall: Lie algebra  $\mathfrak{g}$  has inner derivation Lie 2-algebra  $\mathfrak{g} \xrightarrow{\mu_1} \mathfrak{g}$
- $\mathfrak{h} \xrightarrow{\mu_1} \mathfrak{g}$  lifted to **inner deriv. Lie 3-algebra**  $\mathfrak{h} \xrightarrow{\mu_1} \mathfrak{g} \ltimes \mathfrak{h} \xrightarrow{\mu_1} \mathfrak{g}$

Explicitly:

$$\left( \begin{array}{c} 0 \\ 0 \end{array} \begin{array}{c} \mathfrak{gl}(N, \mathbb{C}) \\ 0 \end{array} \right) \xrightarrow{\mu_1} \left( \begin{array}{c} \mathfrak{u}(N) \\ 0 \end{array} \begin{array}{c} \mathfrak{gl}(N, \mathbb{C}) \\ \mathfrak{u}(N) \end{array} \right) \xrightarrow{\mu_1} \left( \begin{array}{c} \mathfrak{u}(N) \\ 0 \end{array} \begin{array}{c} 0 \\ \mathfrak{u}(N) \end{array} \right)$$

The ABJM model can be completed to a higher gauge theory.

Step 2: Implement the fake curvature conditions

- Here, we are working with a strict **Lie 3-algebra**.
- Gauge potentials:  $A, B, C$ . Curvatures:  $F, H, G$ .
- Conditions  $\mathcal{F} = F - \mu_1(B) = 0$ ,  $\mathcal{H} = H - \mu_1(C) = 0$
- Action:

$$S_{\text{ABJM}} = \int_{\mathbb{R}^{1,2}} \text{tr} \left( \frac{k}{4\pi} \eta A \wedge (dA + \frac{1}{3}[A, A]) \right. \\ \left. - \nabla Z_A^\dagger \wedge * \nabla Z^A - * i \bar{\psi}^A \wedge \not{\nabla} \psi_A \right) + V$$

$$S_{\text{HGTT}} = S_{\text{ABJM}} + \int_{\mathbb{R}^{1,2}} \text{tr} \left( \lambda_1^\dagger \wedge (F - \mu_1(B)) \right. \\ \left. + \lambda_2^\dagger (H - \mu_1(C)) + \lambda_3^\dagger \mu_1(\lambda_2) \right)$$

- This yields **ABJM eoms** + **fake curvature constraints**

There is much more evidence for using higher structures in M-theory.

Proposal for 6d (1,0) models from **tensor hierarchies**

**Samtleben et al., 1108.4060, also 1108.5131**

Gauge structure:

- Graded vector space:  $\mathfrak{g} \xleftarrow{\mathfrak{h}} \mathfrak{h} \xleftarrow{\mathfrak{g}} \mathfrak{l}$
- Maps with structure relations:

$$\mathfrak{h}(\mathfrak{g}(\lambda)) = 0$$

$$\mathfrak{f}(\mathfrak{h}(\chi), \gamma) - \mathfrak{h}(\mathfrak{d}(\mathfrak{h}(\chi), \gamma)) = 0$$

$$\mathfrak{f}(\gamma_{[1}, \mathfrak{f}(\gamma_2, \gamma_3])) - \frac{1}{3} \mathfrak{h}(\mathfrak{d}(\mathfrak{f}(\gamma_{[1}, \gamma_2), \gamma_3])) = 0$$

$$\mathfrak{g}(\mathfrak{b}(\chi_1, \mathfrak{h}(\chi_2))) - 2\mathfrak{d}(\mathfrak{h}(\chi_1), \mathfrak{h}(\chi_2)) = 0$$

$$\mathfrak{g}(\mathfrak{f}^*(\gamma, \lambda) - \mathfrak{d}^*(\mathfrak{h}^*(\lambda), \gamma) + \mathfrak{b}(\mathfrak{g}(\lambda), \gamma)) = 0$$

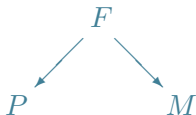
- Relations for a symplectic  $NQ$ -manifold in degrees 1, 2, 3
- Their model is a **higher gauge theory**.
- **No fake curvature condition** is imposed.

How to construct  $(2,0)$ -theories systematically

Using twistor spaces, one can map holomorphic data to solutions to field equations.

Recall the principle of the **Penrose-Ward transform**:

- We construct a double fibration



$P$ : **twistor space**,  $F$ : correspondence space

- $H^n(P, \mathfrak{G})$  (e.g. vector bundles)  $\xleftrightarrow{1:1}$  sols. to field equations.
- Our new contributions:
  - Use (non-abelian) **principal 2-bundles**
  - **New twistor space**
- Can describe in this way:
  - **6d (2,0) superconformal equations of motion**
  - **self-dual strings**

# Known Examples of Twistor Descriptions

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For Yang-Mills theories and its BPS subsectors, there is a wealth of twistor descriptions.

$$\begin{array}{ccc} \mathbb{C}^4 \times \mathbb{C}P^1 & & \\ \swarrow & & \searrow \\ \mathbb{C}P^3_{\circ} & & \mathbb{C}^4 \end{array}$$

Instantons  
hol. vector bundle

$$\begin{array}{ccc} \mathbb{C}^3 \times \mathbb{C}P^1 & & \\ \swarrow & & \searrow \\ T\mathbb{C}P^1 & & \mathbb{C}^3 \end{array}$$

Monopoles  
hol. vector bundle

$$\begin{array}{ccc} \mathbb{C}^{4|12} \times \mathbb{C}P^1 \times \mathbb{C}P^1 & & \\ \swarrow & & \searrow \\ P^{5|6} & & \mathbb{C}^{4|12} \end{array}$$

Super Yang-Mills  
hol. vector bundle

$$\begin{array}{ccc} \mathbb{C}^6 \times \mathbb{C}P^3 & & \\ \swarrow & & \searrow \\ P^6 & & \mathbb{C}^6 \end{array}$$

abelian  $\mathcal{H} = *\mathcal{H}$   
hol. gerbe

Hughston, Murray, Eastwood, CS & M Wolf, Mason et al.

**Note:** last twistor space reduces nicely to the above ones.



New: Penrose-Ward transform for self-dual tensor multiplet.

$$\begin{array}{ccc} & \mathbb{C}^{6|16} \times \mathbb{C}P^3 & \\ & \swarrow \quad \searrow & \\ P^{6|4} & & \mathbb{C}^{6|16} \end{array}$$

non-abelian self-dual tensor multiplet  
hol. principal 2-bundle  
also: semistrict, simplicial, 3-bundles, ...

B Jurčo, CS & M Wolf

Note:

- $P^{6|4}$  is a straightforward SUSY generalization of  $P^6$
- EOMs, abelian:  $\mathcal{H} = \star\mathcal{H}$ ,  $\mathcal{F} = 0$ ,  $\nabla\psi = 0$ ,  $\square\phi = 0$
- $\mathcal{N} = (2,0)$  SC non-abelian tensor multiplet EOMs!
- Reduces search for (2,0)-theory to search for gauge structure

New: Penrose-Ward transform for self-dual strings.

New twistor space parameterizing hyperplanes in  $\mathbb{C}^4$ :

$$\begin{array}{ccc} \mathbb{C}^4 \times \mathbb{C}P^1 \times \mathbb{C}P^1 & & \\ \swarrow & & \searrow \\ P^3 & & \mathbb{C}^4 \end{array}$$

self-dual strings

hol. principal 2-bundle

also: semistrict, simplicial, 3-bundles, ...

B Jurčo, CS & M Wolf

Note:

- **Hyperplane twistor space**  $P^3$ : all of  $\mathcal{O}(1,1) \rightarrow \mathbb{C}P^1 \times \mathbb{C}P^1$
- The spheres  $\mathbb{C}P^1 \times \mathbb{C}P^1$  parameterize an  $\alpha$ - and a  $\beta$ -plane.
- The span of both is a **hyperplane**.
- **Nonabelian** self-dual string equations:  $\mathcal{H} = *d_A\Phi$ ,  $\mathcal{F} = 0$ .
- **Reduces nicely** to the monopole twistor space:  $\mathcal{O}(2) \rightarrow \mathbb{C}P^1$ .

You may be talking about the empty set,  
I want to see some examples...

# A Categorized Nahm Transform

Many ingredients are still missing, more work needs to be done.

- Usually, we wouldn't use **twistors** to construct solutions
- Instead: **ADHM-** and **AHDMN-**constructions
- Equations explicitly **solvable**, **moduli space** under control, etc.
- **No higher Nahm transform** so far
- Substantially more ingredients than twistor construction:
  - **associated 2-vector bundle**
  - **Stringor bundle** (categorized Spinor bundle)
  - **categorized Dirac operator**
- Solutions to these problems: **Major mathematical progress**
- More work is needed...

# Review: The 't Hooft-Polyakov Monopole

The 't Hooft-Polyakov Monopole is a non-singular solution with charge 1.

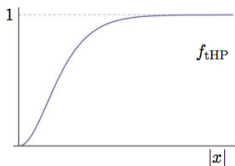
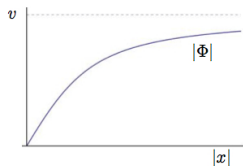
Recall 't Hooft-Polyakov monopole ( $e_i$  generate  $\mathfrak{su}(2)$ ,  $\xi = v|x|$ ):

$$\Phi = \frac{e_i x^i}{|x|^2} (\xi \coth(\xi) - 1), \quad A = \varepsilon_{ijk} \frac{e_i x^j}{|x|^2} \left(1 - \frac{\xi}{\sinh(\xi)}\right) dx^k$$

- At  $S_\infty^2$ :  $\Phi \sim g(\theta)e_3g(\theta)^{-1}$ .  
 $g(\theta) : S_\infty^2 \rightarrow \text{SU}(2)/\text{U}(1)$ : winding 1
- Charge  $q = 1$  with

$$2\pi q = \frac{1}{2} \int_{S_\infty^2} \frac{\text{tr}(F^\dagger \Phi)}{\|\Phi\|} \quad \text{with} \quad \|\Phi\| := \sqrt{\frac{1}{2} \text{tr}(\Phi^\dagger \Phi)}$$

- Higgs field non-singular:



We can write down a non-abelian self-dual string with winding number 1.

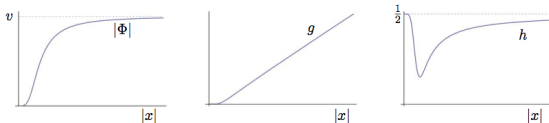
**Self-Dual String** (Lie 2-algebra  $\mathfrak{su}(2) \times \mathfrak{su}(2) \xrightarrow{\mu_1} \mathbb{R}^4$ ,  $\xi = v|x|^2$ ):

$$\Phi = \frac{e_\mu x^\mu}{|x|^3} f(\xi), \quad B_{\mu\nu} = \varepsilon_{\mu\nu\kappa\lambda} \frac{e_\kappa x^\lambda}{|x|^3} g(\xi), \quad A_\mu = \varepsilon_{\mu\nu\kappa\lambda} D(e_\nu, e_\kappa) \frac{x^\lambda}{|x|^2} h(\xi)$$

- Solves indeed  $H = \star \nabla \Phi$  for right  $f(\xi)$ ,  $g(\xi)$ ,  $h(\xi)$
- At  $S_3^\infty$ :  $\Phi \sim g(\theta) \triangleright e_4$ .  $g(\theta) : S_\infty^3 \rightarrow \text{SU}(2)$  has winding 1.
- **Charge**  $q = 1$ :

$$(2\pi)^3 q = \frac{1}{2} \int_{S_3^\infty} \frac{(H, \Phi)}{\|\Phi\|} \quad \text{with} \quad \|\Phi\| := \sqrt{\frac{1}{2}(\Phi, \Phi)},$$

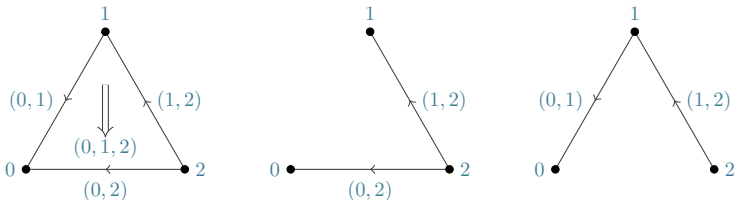
- Higgs field **non-singular**:



# Extensions: Quasi-Categories

Most general, currently available approach: quasi-categories

- Quasi- or  $\infty$ -categories: models for  $(\infty, 1)$ -categories.  
Boardman, Vogt, Joyal
- Example: the **nerve of a category**
- Simplicial sets, which are **Kan complexes**: Horns can be filled:



- Functors**: simplicial maps
- Natural transformations**: simplicial homotopies
- Ševera's differentiation**: yields corresponding  $L_\infty$ -algebra.
- Higher gauge theory: 1602.????, B Jurčo, CS and M. Wolf

# The String Group

A very interesting case: The string group.

- Monopole/instanton solutions: gauge group from **spin group**  
 $\text{Spin}(3) \cong \text{SU}(2)$ ,  $\text{Spin}(4) \cong \text{SU}(2) \times \text{SU}(2)$
- **Higher analogue** of the spin group: **String group**  
Stolz, Teichner, Witten, ...
- Def. via **Whitehead tower** (iteratively delete homotopy groups)

$$\dots \rightarrow \text{String}(n) \rightarrow \text{Spin}(n) \rightarrow \text{Spin}(n) \rightarrow \text{SO}(n) \rightarrow \text{O}(n)$$

- Definition only **up to homotopy**, as a group:  $\infty$ -dimensional
- 2-group models:
  - $\infty$ -dimensional strict 2-group      Baez et al., Nikolaus et al.
  - finite-dimensional quasi 2-group      Schommer-Pries
- Higher gauge theory **1602.????**, G A Demessie and CS
- Conjecture: Gauge 2-group for M5-branes is **String( $E_8$ )**



# Generalized Higher Gauge Theory

Replacing spacetime by a categorified space yields interesting features.

Return to picture from beginning:

Gauge theory

$$a : T[1]M \rightarrow \text{Lie algebra}[1]$$

Higher gauge theory

$$a : T[1]M \rightarrow L_\infty\text{-algebra}[1]$$

Generalize this:

Generalized higher gauge theory  $a : T^*[2]T[1]M \rightarrow L_\infty\text{-algebra}[1]$

Generalized higher gauge theory: P Ritter, CS and L Schmidt

- $T[1]M$  replaced by exact Courant algebroid  $T^*M \oplus TM$
- relation to generalized geometry/double field theory
- $M$  is replaced by a categorified space  $T^*M \rightrightarrows M$
- Gauge connection contains a cov. constant vector field
- Lambert & Papageorgakis suggested eoms for 3-Lie algebra valued tensor multiplet
- Naturally interpreted in generalized higher gauge theory

## Summary:

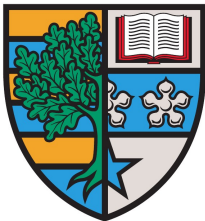
- ✓ Clear **physical and mathematical motivation** to study HGT
- ✓ **Many suggested models** are HGT or GHGT
- ✓ Explicit **higher monopole** and **instanton** solutions
- ✓ Various **twistor constructions** with principal 2-bundles
- ✓ **6d superconformal tensor multiplet equations**

## Future directions:

- ▷ Allow for/study more general **higher groups**
- ▷ Extend and study **twistor constructions**
- ▷ Find **more solutions**
- ▷ Continue translation of higher **ADHM**-constructions

# Looking For the Classical (2,0)-Theory

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