3.36pt

The Self-Dual String and the (2,0)-Theory from Higher Structures

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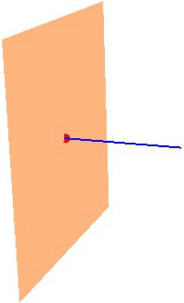
Based on:

- CS & L Schmidt, arXiv:1705.02353, 17??.?????

Motivation: The Dynamics of Multiple M5-Branes

To understand M-theory, an effective description of M5-branes would be very useful.

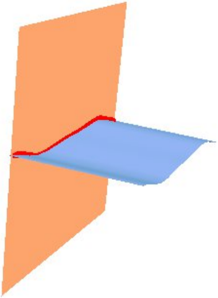


D-branes

- D-branes interact via strings.
- Effective description: theory of endpoints
- Parallel transport of these: Gauge theory
- Study string theory via gauge theory

M5-branes

- M5-branes interact via M2-branes.
- Eff. description: theory of self-dual strings
- Parallel transport: Higher gauge theory
- Long sought $(2,0)$-theory a HGT?



What we know about the (2,0)-theory

Multiple M5-branes are described by a $\mathcal{N} = (2, 0)$ superconformal field theory.

What we know about 6d $\mathcal{N} = (2, 0)$ SCFT:

- String theory considerations: conformal fixed point in 6d
  Witten, Strominger 1995
- Field content: $\mathcal{N} = (2, 0)$ supermultiplet in 6d:
  - a self-dual 3-form field strength
  - five (Goldstone) scalars
  - fermionic partners
- A theory of essentially tensionless light strings
- Supergravity decouples, so study string dynamics separately
- Observables: Wilson surfaces, i.e. parallel transport of strings
- No Lagrangian description known
- As important as $\mathcal{N} = 4$ super Yang-Mills for string theory
- Huge interest in string theory: AGT, AdS$_7$-CFT$_6$, S-duality, ...
- Mathematics: Geom. Langlands, Khovanov Homology, ...

Parallel transport of strings requires category theory

Parallel transport of particles in representation of gauge group G:

- holonomy functor hol : path $\gamma \mapsto \mathsf{hol}(\gamma) \in \mathsf{G}$
- $\mathsf{hol}(\gamma) = P\exp(\int_\gamma A)$, P: path ordering, trivial for $\mathsf{U}(1)$.

Parallel transport of strings with gauge group $\mathsf{U}(1)$:

- map hol : surface $\sigma \mapsto \mathsf{hol}(\sigma) \in \mathsf{U}(1)$
- $\mathsf{hol}(\sigma) = \exp(\int_\sigma B)$, B: connective structure on gerbe.

Nonabelian case:

- definition of surface ordering problematic:
  Eckmann-Hilton argument, rediscovered by physicists
- Way out: 2-categories, Higher Gauge Theory

Need (higher) category theory

Some quotes:

- "We will need to use some very simple notions of category theory, an esoteric subject noted for its difficulty and irrelevance."

  G. Moore and N. Seiberg, 1989
- "We'll only use as much category theory as is necessary. Famous last words..."

  Roman Abramovich
- "Category theory is the subject where you can leave the definitions as exercises."

  John Baez

Objection to a classical (2,0)-theory

Standard objection beyond the previous no-go theorem:

- theory at conformal fixed points $\Rightarrow$ no dimensionful parameter
- fixed points are isolated $\Rightarrow$ no dimensionless parameter
- "No parameters $\Rightarrow$ no classical limit $\Rightarrow$ no Lagrangian."

Answers:

- Same arguments for M2-brane Schwarz, 2004
- There, integer parameters arose from orbifold $\mathbb{R}^8/\mathbb{Z}_k$
- Same should happen for M5-branes
- Even if no Lagrangian, BPS-states may exist classically $\Rightarrow$ "self-dual strings"
- Even if not, study quantum features of related theories.

Additional Motivation

Focus on Self-Dual Strings, BPS states in (2,0)-theory.

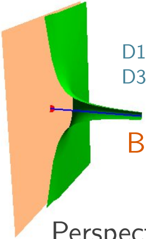
Observations:

- Lift of D-brane interpretation of BPS monopoles to M-theory
- Involves "categorified" of "higher" version of gauge theory

Additional reasons for studying self-dual strings:

- Categorified Integrability
  - Twistor descriptions developed CS, Martin Wolf 2012-2016
  - Categorified Nahm Transform ⇒ Categorified Dirac operator
- Involves a higher quantization of S^3
  - Important for non-geometric backgrounds in string theory
- Examples of categorified/higher principal bundles
  - Important for mathematical progress

The non-abelian self-dual string



	0	1	2	3	4	5	6
D1	×						×
D3	×	×	×	×			

BPS configuration

Perspective of D3:

Bogomolny monopole eqn.

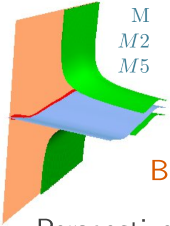
$F = \nabla^2 = *\nabla\Phi$ on $\mathbb{R}^3$

$\updownarrow$ Nahm transform $\updownarrow$

Perspective of D1:

Nahm eqn.

$$\frac{\mathrm{d}}{\mathrm{d}x^6}X^i + \varepsilon^{ijk}[X^j, X^k] = 0$$



M	0	1	2	3	4	5	6
$M2$	×					×	×
$M5$	×	×	×	×	×	×	

BPS configuration

Perspective of M5:

Abelian Self-dual string eqn.

$H := \mathrm{d}B = *\mathrm{d}\Phi$ on $\mathbb{R}^4$

$\updownarrow$ genlzd. Nahm transform (?) $\updownarrow$

Perspective of M2:

Hoppe-Basu-Harvey eqn. (??)

$$\frac{\mathrm{d}}{\mathrm{d}x^6}X^\mu + \varepsilon^{\mu\nu\rho\sigma}[X^\nu, X^\rho, X^\sigma] = 0$$

What is a non-abelian self-dual string?

Recall:

- Abelian Dirac Monopole: singular on $\mathbb{R}^3$
- Non-abelian 't Hooft–Polyakov Monopole: non-singular on $\mathbb{R}^3$
- Abelian Dirac Monopole: can add solutions (non-interacting)
- Abelian Self-Dual String: singular on $\mathbb{R}^4$
- Abelian Self-Dual String: can add solutions (non-interacting)

Goal: Non-abelian self-dual string with

- non-singular solution on $\mathbb{R}^4$
- interacting solution

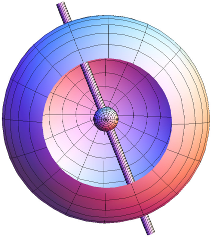
Steps:

- Identity gauge structure
- Identify equations of motion
- Find at least elementary (charge 1) solution

Identifying gauge structure: Monopoles

Monopoles

- Solution to Bogomolny eqn. $F := \nabla^2 = *\nabla\phi$
- Abelian: singular on $\mathbb{R}^3$, Dirac strings
- Principal bundle over S^2
- Non-Abelian: non-singular on $\mathbb{R}^3$



$$\begin{tikzcd}
\mathsf{U}(1) \arrow[r, hook] \arrow[rr, dashed, bend left, "\rho"] & \mathsf{SU}(2) \cong S^3 \arrow[d, "\pi"] \arrow[rr, dashed, bend left, "\pi\times\mathrm{id}"] & \mathsf{SU}(2) \arrow[r, hook] & S^2 \times \mathsf{SU}(2) \arrow[d, "\mathrm{pr}"] \\
 & S^2 \arrow[rr, dashed, bend left, "\mathrm{id}"] & & S^2
\end{tikzcd}$$

abelian, Dirac

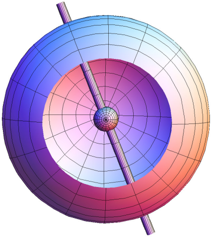
non-Abelian, ’t Hooft-Polyakov

$\Rightarrow$ Choose $\mathsf{SU}(2)$, as trivialization possible.

Identifying gauge structure: Self-Dual Strings

Self-Dual Strings ("higher monopoles")

- Abelian: singular on $\mathbb{R}^4$, Dirac strings
- Solution to $H := \mathrm{d}B = *\mathrm{d}\phi$
- Gerbe over S^3
- Non-Abelian: ?



$$\begin{tikzcd}
\mathsf{BU}(1) \arrow[r, hook] \arrow[rrr, dashed, bend left, "\rho"] & \mathscr{G}_{\mathrm{F}} \arrow[d, "\pi"] \arrow[rrr, dashed, bend left, "\pi\times\mathrm{id}"] & & \mathscr{G}_{\mathrm{F}} \arrow[r, hook] & (S^3 \rightrightarrows S^3) \times \mathscr{G}_{\mathrm{F}} \arrow[d, "\mathrm{pr}"] \\
& (S^3 \rightrightarrows S^3) \arrow[rrr, dashed, bend left, "\mathrm{id}"] & & & (S^3 \rightrightarrows S^3)
\end{tikzcd}$$

abelian

non-Abelian ?

$\Rightarrow$ Choose $\mathscr{G}_F$, with 2-group structure: String 2-group

(many other reasons for this)

The String Group

- Monopole/instanton solutions: gauge group from spin group
  $\mathsf{Spin}(3) \cong \mathsf{SU}(2)$, $\mathsf{Spin}(4) \cong \mathsf{SU}(2) \times \mathsf{SU}(2)$
- Higher analogue of the spin group: String group
  Stolz, Teichner, Witten, ...
- Def. via Whitehead tower (iteratively delete homotopy groups)

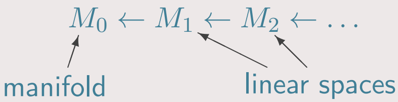
$$\ldots \to \mathsf{String}(n) \to \mathsf{Spin}(n) \to \mathsf{Spin}(n) \to \mathsf{SO}(n) \to \mathsf{O}(n)$$

- Definition only up to homotopy, as a group: ∞-dimensional
- 2-group models:
  - ∞-dimensional strict 2-group BCCS (2005)
  - finite-dimensional quasi 2-group Schommer-Pries (2009)
  - other 2-group models, e.g. Nikolaus et al. ...
- Higher gauge theory developed Demessie, CS (2016)
- Many reasons: Gauge 2-group for M5-branes is $\mathsf{String}(E_8)$
  Aschieri, Jurco (2004)

NQ-Manifolds

N-manifolds, NQ-manifold

- $\mathbb{N}_0$-graded manifold with coordinates of degree $0, 1, 2, \ldots$



$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots$$

- NQ-manifold: vector field Q of degree 1, $Q^2 = 0$
- Physicists: think ghost numbers, BRST charge, SFT

Examples:

- Tangent algebroid $T[1]M$, $\mathcal{C}^\infty(T[1]M) \cong \Omega^\bullet(M)$, $Q = \mathrm{d}$
- Lie algebra $\mathfrak{g}[1]$, coordinates ξ^a of degree 1:

$$Q = -\tfrac{1}{2} f^c_{ab} \xi^a \xi^b \frac{\partial}{\partial \xi^c} \quad , \qquad \text{Jacobi identity} \Leftrightarrow \;\; Q^2 = 0$$

L_∞-Algebras, Lie 2-Algebras

Lie n-algebroid, a special kind of L_∞-algebroid:

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Lie n-algebra or n-term L_∞-algebra:

$$* \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Important example: Lie 2-algebra

- Graded vector space: $W[1] \leftarrow V[2]$
- Coordinates: w^a of degree 1 on $W[1]$, v^i of degree 2 on $V[2]$
- Most general vector field Q of degree 1:

$$Q = -m_i^a v^i \frac{\partial}{\partial w^a} - \tfrac{1}{2} m_{ab}^c w^a w^b \frac{\partial}{\partial w^c} - m_{ai}^j w^a v^i \frac{\partial}{\partial v^j} - \tfrac{1}{3!} m_{abc}^i w^a w^b w^c \frac{\partial}{\partial v^i}$$

- Induces “brackets”/“higher products”:

$$\mu_1(\tau_i) = m_i^a \tau_a \ , \quad \mu_2(\tau_a, \tau_b) = m_{ab}^c \tau_c \ , \quad \ldots \ , \quad \mu_3(\tau_a, \tau_b, \tau_c) = m_{abc}^i \tau_i$$

- $Q^2 = 0 \Leftrightarrow$ Homotopy Jacobi identities, e.g. $\mu_1(\mu_1(-)) = 0$
- Failure of Jacobi identity: $\mu_2(x, \mu_2(y, z)) + \ldots = \mu_1(\mu_3(x, y, z))$

String Lie 2-algebra

- Recall: $\mathscr{G}_F$ can be extended to String 2-group model
- Lie differentiate (e.g. Demessie, CS (2016))
- Result: String Lie 2-algebra $\mathfrak{string}(3) = \mathbb{R}[1] \rightarrow \mathfrak{su}(2)$ with

$$Q\xi^\alpha = -\tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma~, \quad Qb = -\tfrac{1}{3!} f_{\alpha\beta\gamma}\xi^\alpha\xi^\beta\xi^\gamma~.$$

or

$$\mu_2(x_1,x_2) = [x_1,x_2]~, \quad \mu_3(x_1,x_2,x_3) = (x_1,[x_2,x_3])$$

where $x_{1,2,3} \in \mathfrak{su}(2)$.

Gauge Theory from NQ-manifolds

- Ideas: Atiyah, Strobl et al., Sati, Schreiber, Stasheff
- Recall: Chevalley-Eilenberg algebra of Lie algebra $\mathfrak{g}$:

$$\mathrm{CE}(\mathfrak{g}) = \mathcal{C}^\infty(\mathfrak{g}[1]) \ , \quad Q\xi^\alpha = -\tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma$$

- Double to Weil algebra

$$\mathrm{W}(\mathfrak{g}) := \mathcal{C}^\infty(T[1]\mathfrak{g}[1]) \ , \quad Q = Q_{\mathrm{CE}} + \sigma \ , \quad \sigma Q_{\mathrm{CE}} = -Q_{\mathrm{CE}}\sigma$$

- Potentials/curvatures/Bianchi identities from dga-morphisms

$$(A,F) : \mathrm{W}(\mathfrak{g}) \to \Omega^\bullet(M) = W(M)$$

$$\xi^\alpha \mapsto A^\alpha$$

$$(\sigma\xi^\alpha) = Q\xi^\alpha + \tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma \mapsto F^\alpha = (\mathrm{d}A + \tfrac{1}{2}[A,A])^\alpha$$

$$Q(\sigma\xi^\alpha) = -f^\alpha_{\beta\gamma}(\sigma\xi^\alpha)\xi^\beta \mapsto (\nabla F)^\alpha = 0$$

- Gauge transformations: homotopies between dga-morphisms
- Topological invariants: invariant polynomials in $\mathrm{W}(\mathfrak{g})$

Higher Gauge Theory from NQ-manifolds

- Recall: Chevalley-Eilenberg algebra of String Lie 2-algebra $\mathfrak{g}$:

$$\mathrm{CE}(\mathfrak{g}) = \mathcal{C}^\infty(\mathbb{R}[2] \to \mathfrak{su}(2)[1])~,$$

$$Q\xi^\alpha = -\tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma \quad \text{and} \quad Qb = \tfrac{1}{3!} f_{\alpha\beta\gamma}\xi^\alpha\xi^\beta\xi^\gamma~.$$

- Double to Weil algebra

$$\mathrm{W}(\mathfrak{g}) := \mathcal{C}^\infty(T[1]\mathfrak{g}[1])~,\quad Q = Q_{\mathrm{CE}} + \sigma~,\quad \sigma Q_{\mathrm{CE}} = -Q_{\mathrm{CE}}\sigma$$

- Potentials/curvatures/Bianchi identities from dga-morphisms

$$(A,B,F,H) : \mathrm{W}(\mathfrak{g}) \to \Omega^\bullet(M) = W(M)$$

$$\xi^\alpha \mapsto A^\alpha \in \Omega^1(M) \quad \text{and} \quad b \mapsto B \in \Omega^2(M)$$

$$(\sigma\xi^\alpha) = Q\xi^\alpha + \tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma \mapsto F^\alpha = (\mathrm{d}A + \tfrac{1}{2}[A,A])^\alpha$$

$$(\sigma b) = Qb - \tfrac{1}{3!} f_{\alpha\beta\gamma}\xi^\alpha\xi^\beta\xi^\gamma \mapsto H = \mathrm{d}B - \tfrac{1}{3!}(A,[A,A])$$

- Bianchi identities: $\nabla F = 0$ and $\mathrm{d}H = -\frac{1}{2}(\mathrm{d}A,[A,A])$
- Gauge trafos and Top. invariants derived as above

Derive equations of motion

Higher Gauge potential for $\mathfrak{string}(3)$: CS, Schmidt (2017)

$$A \in \Omega^1(\mathbb{R}^4) \otimes \mathfrak{su}(2) \ , \quad B \in \Omega^2(\mathbb{R}^4) \otimes \mathbb{R} \ ,$$

Add Higgs field:

$$\phi \in \Omega^0(\mathbb{R}^4) \otimes \mathbb{R}$$

Equations of motion:

- Straightforward discussion from dga-morphisms yields:

$$H = \mathrm{d}B - \tfrac{1}{3!}(A, [A, A]) = *\nabla\phi \ , \quad F = \mathrm{d}A + \tfrac{1}{2}[A, A] = 0 \ .$$

These equations, however, suffer from many problems.

Solution:

- 10d heterotic SUGRA: coupling A requires modification of H
  Bergshoeff et al. (1982), Chaplin & Manton (1983)
- Also: Anomaly cancellation condition $\mathrm{d}H = (F, F) = \frac{1}{2}p_1$
- Mathematically: (Twisted) String Structure
  Sati, Schreiber, Stasheff (2009)

Derive equations of motion

Field Content with values in $\mathfrak{string}(3)$:

$$A \in \Omega^1(\mathbb{R}^4) \otimes \mathfrak{su}(2)\ , \quad B \in \Omega^2(\mathbb{R}^4) \otimes \mathbb{R}\ , \quad \phi \in \Omega^0(\mathbb{R}^4) \otimes \mathbb{R}$$

Use categorical equivalences:

- $\mathsf{SU}(N)$-bundle: $\mathsf{U}(N)$-bundle + trivialization of $\mathsf{U}(1)$-factor
- $\mathsf{String}(3)$-bundle as $\mathsf{Spin}(3)$-bundle + $\mathsf{BBU}(1)$-factor

  Sati, Schreiber, Stasheff (2009)
- Upshot: add (A, F) to H, which yields:

$$H = \mathrm{d}B + \tfrac{1}{2}(A, \mathrm{d}A) + \tfrac{1}{3!}(A, [A, A]) = *\mathrm{d}\phi \;\Rightarrow\; dH = (F, F) = *\Box\phi$$

- ϕ should “know” all about configuration, thus demand also

$$F = \mathrm{d}A + \tfrac{1}{2}[A, A] = *F$$

- Check 1: Nice reduction of 2nd equation to monopoles on $\mathbb{R}^3$
- First equation requires little more work, also reduces perfectly
- Check 2: BPS equations for (1,0)-model (more later)

Bogomolny trick

Bogomolny's trick familiar from instanton/monopoles works, too:

$$S=\int_{\mathbb{R}^4} H\wedge *H+\mathrm{d}\varphi\wedge *\mathrm{d}\varphi+(F,*F)$$

Rewrite:

$$S=\int_{\mathbb{R}^4}(H-*\mathrm{d}\varphi)\wedge *(H-*\mathrm{d}\varphi)+2H\wedge\mathrm{d}\varphi+ \\ \tfrac{1}{2}\big((F-*F),*(F-*F)\big)+\big(F,F\big)$$

Minimum/Bogomolny bound:

$$H=*\mathrm{d}\varphi\ ,\quad F=*F$$

Topological invariants from minimum of action:

$$S_{\min}=2\int_{\mathbb{R}^4}H\wedge\mathrm{d}\varphi+\int_{\mathbb{R}^4}\big(F,F\big)=2\int_{\mathbb{R}^4}H\wedge\mathrm{d}\varphi+\int_{S^3_\infty}H$$

Solution

Equations of motion:

$$F = \mathrm{d}A + \tfrac{1}{2}[A,A] = *F \quad \text{and} \quad H = \mathrm{d}B + \mathrm{CS}(A) = *\mathrm{d}\phi$$

Solution:

$$A_\mu(x) = \frac{1}{\mathrm{i}} \frac{\eta^i_{\mu\nu}\,\sigma_i\,(x-x_0)^\nu}{\rho^2 + (x-x_0)^2}\ , \quad B = 0\ , \quad \varphi = \frac{((x-x_0)^2 - 2\rho^2}{\big((x-x_0)^2+\rho^2\big)^2}$$

cf. also Akyol, Papadopoulos 2012

Consistency checks:

- Globally non-singular on $\mathbb{R}^4$
- Approaches abelian solution $\frac{1}{x^2}$ as $x \to \infty$
- Moduli: same as instanton:
  - Position
  - Size (conformal symmetry of $F = *F$)
  - Residual global $\mathrm{SU}(2)$ gauge symmetry

Consistency check: BCSS String 2-Group Model

Alternative, but equivalent form of String Lie 2-algebra:

$$\mathfrak{string}_{\hat{\Omega}} = \Big(\Omega\mathfrak{su}(2) \oplus \mathbb{R} \to P_0\mathfrak{su}(2) \Big)$$

- $\Omega\mathfrak{su}(2)$ and $P_0\mathfrak{su}(2)$: based loop and path spaces
- Quasi-Isomorphism between both models
- Should be reflected in equivalence of physics

We find:

- Fields: $A \in \Omega^1(\mathbb{R}^4) \otimes P_0\mathfrak{su}(2)$, $B \in \Omega^2(\mathbb{R}^4) \otimes (\Omega\mathfrak{su}(2) \oplus \mathbb{R})$ and Higgs field $\varphi \in \Omega^0(\mathbb{R}^4) \otimes (\Omega\mathfrak{su}(2) \oplus \mathbb{R})$
- Equations of motion, modified for twisted string structures:

$$\mathcal{F} := \mathrm{d}A + \tfrac{1}{2}\mu_2(A, A) + \mu_1(B)$$
$$H := \mathrm{d}B + \mu_2(A, B) - \kappa(A, \mathcal{F}) = *\nabla\varphi$$

- Explicit 1:1-map between gauge equivalence classes
- $\Rightarrow$ Physics respects categorical equivalence

Extension from BPS to (1,0)-theory

BPS states work, what about classical (2,0)-theory?

Recall:

- 6d (1,0)-model derived from tensor hierarchies
  Samtleben, Sezgin, Wimmer (2011)
- Issue 1: Choice of gauge structure unclear
- Issue 2: cubic interactions
- Issue 3: scalar fields with wrong sign kinetic term
- Issue 4: Self-duality of 3-form imposed by hand

Previous observation:

- Gauge structure has underlying Lie 3-algebra + extra struct.
  Palmer, CS (2013), Samtleben et al. (2014)

From $\mathfrak{string}(3)$ to a (1,0) gauge structure

New: Schmidt, CS (2017)

- Idea: use $\mathfrak{string}(3)$ as gauge structure in this model
- Issue: need suitable notion of inner product for action
- Appropriate inner products on L_∞-algebras: symplectic forms
- Consequence: Double twisted $\mathfrak{string}(3)$

  $$\mathbb{R} \to \mathbb{R} \to \mathfrak{su}(2)$$

  to something symplectic:

  $$\mathbb{R} \oplus \mathfrak{su}(2)^* \to \mathbb{R} \oplus \mathbb{R}^* \to \mathfrak{su}(2) \oplus \mathbb{R}^*$$
- This carries natural inner product
- Can be extended to Lie 3-algebra
- Has necessary extra structure

Properties of resulting (1,0)-theory

Field content:

- (1,0) tensor multiplet (ϕ, χ^i, B), values in $\mathbb{R}^2$, $\phi = \phi_s + \phi_r$, ...
- (1,0) vector multiplet (A, λ^i, Y^{ij}), values in $\mathfrak{su}(2) \oplus \mathbb{R}$
- C-field, values in $\mathbb{R} \oplus \mathfrak{su}(2)^*$

Action (schematically):

$$\begin{aligned} S = \int_{\mathbb{R}^{1,5}} \Big(& \mathcal{H} \wedge *\mathcal{H} + \mathrm{d}\phi \wedge *\mathrm{d}\phi - *\langle \bar{\chi}, \not{\partial} \chi \rangle + \mathcal{H}_s \wedge *\langle \bar{\lambda}, \gamma_{(3)} \lambda \rangle + *\langle Y, \bar{\lambda} \rangle \chi_s \\ & + \phi_s \big((\mathcal{F}, *\mathcal{F}) - *(Y, Y) + *(\bar{\lambda}, \not{\nabla} \lambda) \big) + \langle \bar{\lambda}, \mathcal{F} \rangle \wedge *\gamma_{(2)} \chi_s \\ & + \mu_1(C) \wedge \mathcal{H}_s + B_s \wedge (\mathcal{F}, \mathcal{F}) + B_s \wedge ([A, A], [A, A]) \Big) \end{aligned}$$

This solves problems 1 and 2:

- Choice of gauge structure for ADE-(2,0)-theories clear.
- No cubic interaction term for scalar fields

Consistency check: Reduction to SYM theory

Crucial consistency check: Reduction to D-branes/SYM theory

$$\begin{aligned} S = \int_{\mathbb{R}^{1,5}} \Big(& \mathcal{H} \wedge *\mathcal{H} + \mathrm{d}\phi \wedge *\mathrm{d}\phi - *\langle \bar{\chi}, \not{\partial}\chi \rangle + \mathcal{H}_s \wedge *\langle \bar{\lambda}, \gamma_{(3)}\lambda \rangle + *\langle Y, \bar{\lambda} \rangle \chi_s \\ & + \phi_s \big((\mathcal{F}, *\mathcal{F}) - *(Y,Y) + *(\bar{\lambda}, \not{\nabla}\lambda) \big) + \langle \bar{\lambda}, \mathcal{F} \rangle \wedge *\gamma_{(2)}\chi_s \\ & + \mu_1(C) \wedge \mathcal{H}_s + B_s \wedge (\mathcal{F}, \mathcal{F}) + B_s \wedge ([A,A],[A,A]) \Big) \end{aligned}$$

- Compactify along x^{10}, x^9 interpret as $\langle \phi_s \rangle = \frac{1}{\pi^2 R_{10} R_9} = \frac{1}{\pi^2 R_{10}^2}$
- Strong coupling expansion around $\langle \phi_s \rangle$ (cf. M2 → D2)
- Coupling constants:

$$\int_{T^2} \mathrm{dvol}(T^2) \frac{1}{16\pi^2 R_{10}^2} = \frac{4\pi^2 R^9 R^{10}}{16\pi^2 R_{10}^2} = \frac{1}{4} \frac{R_9}{R_{10}} = \frac{1}{4 g_s} = \frac{1}{4 g_{\mathrm{YM}}^2}$$

- This yields 4d $\mathcal{N} = 2$ SYM with gauge group SU(2)!

Conclusions

Summary and Outlook.

Summary:

- Categorified structures appear naturally in M-theory
- Higher analogue of $\mathsf{SU}(2)$ is $\mathsf{String}(3)$
- There is a non-abelian self-dual string
- Better understanding of (1,0)-theory
- Dimensional reduction to $\mathcal{N} = 4$ SYM works!

Soon to come:

- ▷ Interpretation of fuzzy S^3
- ▷ Categorified Integrability
- ▷ Access new superconformal field theories
- ▷ M5-brane models?

The Self-Dual String and the (2,0)-Theory from Higher Structures

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