Matrix Models and D-Branes in Twistor String Theory

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Based on:

- JHEP **0603** (2006) 002, O. Lechtenfeld and CS.

Well-known motivation for studying twistor strings:

- Alternative description of the AdS/CFT correspondence
- New tools for calculating gluon scattering amplitudes
- Alternative descriptions of supergravity

My motivation here:

- Description of super D-branes?
- Relationship between topological and physical D-branes?
- Rôle of Calabi-Yau supermanifolds in mirror symmetry?

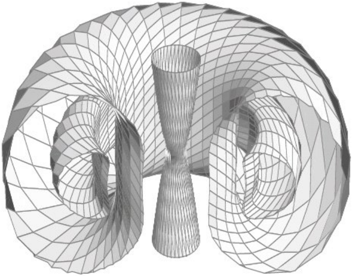
$\Rightarrow$ Study variations of the usual twistor geometries and the associated Penrose-Ward transform.

Here: Full dimensional reductions yielding matrix models with interesting interpretations in terms of D-branes.

The presented results are only a *very* preliminary step towards answering the above questions.

Outline

1. Notation: Twistors and Penrose-Ward transform
2. Construction of the matrix models
3. D-Brane interpretation and completion for
   - ADHM construction
   - Nahm construction
4. Conclusions



The Twistor Correspondence

The twistor correspondence is a relation between subsets of twistor space and spacetime.

Incidence Relation: $\omega^\alpha = x^{\alpha\dot\alpha}\lambda_{\dot\alpha}$, Twistor: $Z^i = (\omega^\alpha, \lambda_{\dot\alpha}) \in \mathbb{C}P^3$

Twistor Correspondence

Point $x^{\alpha\dot\alpha}$ corresponds to sphere $\mathbb{C}P^1 \ni \lambda_{\dot\alpha}$

A twistor Z^i is incident to a plane of points $x^{\alpha\dot\alpha} = x_0^{\alpha\dot\alpha} + \kappa^\alpha\lambda^{\dot\alpha}$.

Decompactification

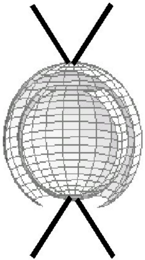
$\mathbb{C}P^3$	is the twistor space of	S^4 or S^4_c
$\mathbb{C}P^1$	take out	∞
$\mathcal{P}^3$	is the twistor space of	$\mathbb{R}^4$ or $\mathbb{C}^4$

$\mathbb{C}P^1_\infty$ is described by $\lambda_{\dot\alpha} = 0$, therefore:

$$\mathcal{P}^3 := \mathcal{O}(1) \oplus \mathcal{O}(1) \to \mathbb{C}P^1$$

Homog. coords. $\lambda_{\dot\alpha}$ on $\mathbb{C}P^1$ and ω^α in fibres

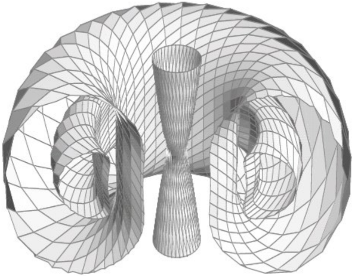
Moduli of sections of $\mathcal{P}^3$: $x^{\alpha\dot\alpha} \in \mathbb{C}^4$

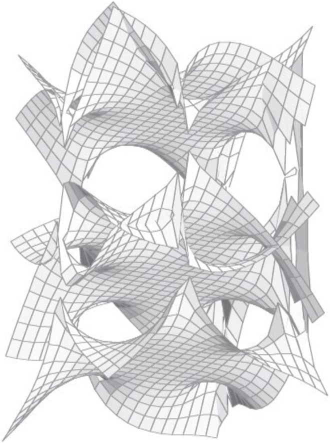


Underlying Idea of Twistor String Theory

To make contact with string theory, we need to extend this picture supersymmetrically.

Marrying Twistor- and Calabi-Yau geometry





... with supermanifolds: Witten, hep-th/0312171

Supertwistor Space

The supertwistor space $\mathcal{P}^{3|\mathcal{N}}$ is a holomorphic vector bundle of rank $3|4\mathcal{N}$ over $\mathbb{C}P^1$.

The Supertwistor Space $\mathcal{P}^{3|\mathcal{N}}$

Start from $\mathbb{C}P^{3|\mathcal{N}}$, take out $\mathbb{C}P^{1|\mathcal{N}}$ at infinity:

$$\mathcal{P}^{3|\mathcal{N}} := \mathbb{C}^2 \otimes \mathcal{O}(1) \oplus \mathbb{C}^{\mathcal{N}} \otimes \Pi\mathcal{O}(1) \to \mathbb{C}P^1$$

Incidence Relations

$$\omega^\alpha = x^{\alpha\dot{\alpha}}\lambda_{\dot{\alpha}}$$

$$\eta_i = \eta_i^{\dot{\alpha}}\lambda_{\dot{\alpha}}$$

Double Fibration

$$\begin{array}{ccc} & \mathbb{C}^{4|2\mathcal{N}} \times \mathbb{C}P^1 & \\ \swarrow & & \searrow \\ \mathcal{P}^{3|\mathcal{N}} & & \mathbb{C}^{4|2\mathcal{N}} \end{array}$$

First Chern Class of $\mathcal{P}^{3|4}$

$T\mathbb{C}P^1$ 2, $\mathcal{O}(1)$ 1, $\Pi\mathcal{O}(1)$ -1, in total: $c_1 = 0$.
Therefore, there exists a holomorphic measure $\Omega^{3,0|4,0}$.

Outline of the Penrose-Ward Transform on $\mathcal{P}^{3|4}$

The PW-transform takes us from the topological B-model to SDYM theory.

topological B-model on $\mathcal{P}^{3|4}$

$$\Updownarrow$$

holomorphic Chern-Simons theory on $\mathcal{E} \to \mathcal{P}^{3|4}$:

$$\int \Omega^{3,0|4,0} \wedge \operatorname{tr}\left(\mathcal{A}^{0,1} \wedge \bar{\partial}\mathcal{A}^{0,1} + \tfrac{2}{3}\mathcal{A}^{0,1} \wedge \mathcal{A}^{0,1} \wedge \mathcal{A}^{0,1}\right)$$

with eom $\bar{\partial}\mathcal{A}^{0,1} + \mathcal{A}^{0,1} \wedge \mathcal{A}^{0,1} = 0$

$$\Updownarrow$$

holomorphic vector bundles over $\mathcal{P}^{3|4}$

$$\Updownarrow$$

solutions to the $\mathcal{N} = 4$ SDYM equations on $\mathbb{C}^{4|8}$

Field contents: $(f_{\alpha\beta}, \chi^{\alpha i}, \phi^{[ij]}, \tilde{\chi}_{\dot{\alpha}}^{[ijk]}, G_{\dot{\alpha}\dot{\beta}}^{[ijkl]})$

$$f_{\dot{\alpha}\dot{\beta}} = 0\ , \qquad \nabla_{\alpha\dot{\alpha}}\tilde{\chi}^{\dot{\alpha}ijk} - [\chi_\alpha^{[i}, \phi^{jk]}] = 0\ ,$$

$$\nabla_{\alpha\dot{\alpha}}\chi^{\alpha i} = 0\ , \qquad \varepsilon^{\dot{\alpha}\dot{\gamma}}\nabla_{\alpha\dot{\alpha}}G_{\dot{\gamma}\dot{\delta}}^{[ijkl]} + \ldots = 0\ .$$

$$\Box\phi^{ij} + 2\{\chi^{\alpha i}, \chi_\alpha^j\} = 0\ ,$$

Penrose-Ward Transform on $\mathcal{P}^{3|4}_{\tau}$

Imposing reality conditions simplifies the situation significantly.

Introducing a real structure, the double fibration collapses:

$$\begin{array}{ccc} & \mathbb{C}^{4|2\mathcal{N}} \times \mathbb{C}P^1 & \\ \swarrow & & \searrow \\ \mathcal{P}^{3|\mathcal{N}} & & \mathbb{C}^{4|2\mathcal{N}} \end{array} \quad \longrightarrow \quad \mathcal{P}^{3|\mathcal{N}}_{\tau} \to \mathbb{R}^{4|2\mathcal{N}}_{\tau}$$

($\tau_{\pm 1}$ related to Kleinian and Euclidean metrics on $\mathbb{R}^{4|2\mathcal{N}}_{\tau}$.)

Now: Field expansion of hCS gauge potential $\mathcal{A}^{0,1}$ available:

$$\begin{aligned} \mathcal{A}_{\alpha} =& \lambda^{\dot\alpha}\, A_{\alpha\dot\alpha}(x) + \eta_i \chi^i_{\alpha}(x) + \gamma\, \tfrac{1}{2!}\, \eta_i\eta_j\, \hat\lambda^{\dot\alpha}\, \phi^{ij}_{\alpha\dot\alpha}(x) + \\ & \gamma^2\, \tfrac{1}{3!}\, \eta_i\eta_j\eta_k\, \hat\lambda^{\dot\alpha}\, \hat\lambda^{\dot\beta}\, \tilde\chi^{ijk}_{\alpha\dot\alpha\dot\beta}(x) + \gamma^3\, \tfrac{1}{4!}\, \eta_i\eta_j\eta_k\eta_l\, \hat\lambda^{\dot\alpha}\, \hat\lambda^{\dot\beta}\, \hat\lambda^{\dot\gamma}\, G^{ijkl}_{\alpha\dot\alpha\dot\beta\dot\gamma}(x) \\ \mathcal{A}_{\bar\lambda} =& \gamma^2 \eta_i\eta_j\, \phi^{ij}(x) - \gamma^3 \eta_i\eta_j\eta_k\, \hat\lambda^{\dot\alpha}\, \tilde\chi^{ijk}_{\dot\alpha}(x) + 2\gamma^4 \eta_i\eta_j\eta_k\eta_l\, \hat\lambda^{\dot\alpha}\, \hat\lambda^{\dot\beta} G^{ijkl}_{\dot\alpha\dot\beta}(x) \end{aligned}$$

Popov, CS, ATMP 9 (2005) 931

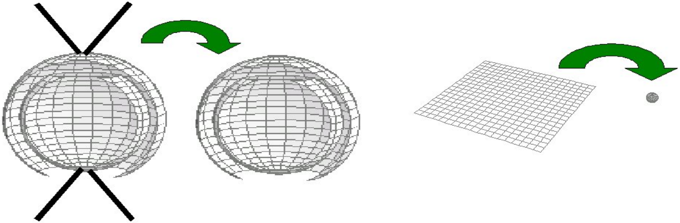
This field expansion makes the equivalence hCS$\leftrightarrow$ SDYM manifest.

Matrix Models

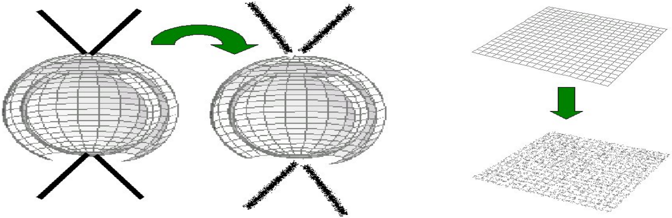
Matrix models are obtained by dim. reduction or from spacetime noncommutativity.

Two ways of obtaining the matrix models:

- Dimensionally reducing the moduli space $\mathbb{R}^{4|8} \to \mathbb{R}^{0|8}$:



- Making the moduli space $\mathbb{R}^{4|8}$ noncommutative:



Matrix Models via Dimensional Reduction

Full dimensional reduction yields equivalence between SDYM MM and hCS MQM.

- Matrix Model from $\mathcal{N}=4$ SDYM theory:

$$S := \mathrm{tr}\left(G^{\dot\alpha\dot\beta}\left(-\tfrac{1}{2}\varepsilon^{\alpha\beta}[A_{\alpha\dot\alpha}, A_{\beta\dot\beta}]\right) + \tfrac{\varepsilon}{2}\phi^{ij}[A_{\alpha\dot\alpha},[A^{\alpha\dot\alpha},\phi_{ij}]] + \dots \right)$$

- Matrix Model from $\mathcal{N}=4$ hCS theory (MQM):

$$S := \int_{\mathbb{C}P^1_{\mathrm{ch}}} \Omega_{\mathrm{red}} \wedge \mathrm{tr}\,\varepsilon^{\alpha\beta}\mathcal{X}_\alpha\left(\bar\partial\mathcal{X}_\beta + \left[\mathcal{A}^{0,1}_{\mathbb{C}P^1}, \mathcal{X}_\beta\right]\right)$$

$$\Omega_{\mathrm{red}} := \Omega^{3,0|4,0}|_{\mathbb{C}P^1_{\mathrm{ch}}} \qquad \Omega_{\mathrm{red}\pm} = \pm \mathrm{d}\lambda_\pm \wedge \mathrm{d}\eta_1^\pm \dots \mathrm{d}\eta_4^\pm$$

- Equivalence explicitly via:

$$\mathcal{X}_\alpha = \lambda^{\dot\alpha} A_{\alpha\dot\alpha} + \eta_i \chi^i_\alpha + \gamma\,\tfrac{1}{2!}\,\eta_i\eta_j\,\hat\lambda^{\dot\alpha}\,\phi^{ij}_{\alpha\dot\alpha} + \gamma^2\,\tfrac{1}{3!}\,\eta_i\eta_j\eta_k\,\hat\lambda^{\dot\alpha}\,\hat\lambda^{\dot\beta}\,\tilde\chi^{ijk}_{\alpha\dot\alpha\dot\beta} + \gamma^3\,\tfrac{1}{4!}\,\eta_i\eta_j\eta_k\eta_l\,\hat\lambda^{\dot\alpha}\,\hat\lambda^{\dot\beta}\,\hat\lambda^{\dot\gamma}\,G^{ijkl}_{\alpha\dot\alpha\dot\beta\dot\gamma}$$

$$\mathcal{A}_{\bar\lambda} = \gamma^2\eta_i\eta_j\,\phi^{ij} - \gamma^3\eta_i\eta_j\eta_k\,\hat\lambda^{\dot\alpha}\,\tilde\chi^{ijk}_{\dot\alpha} + 2\gamma^4\eta_i\eta_j\eta_k\eta_l\,\hat\lambda^{\dot\alpha}\,\hat\lambda^{\dot\beta}G^{ijkl}_{\dot\alpha\dot\beta}$$

Matrix Models from Noncommutativity

Functions on the noncommutative moduli space are infinite-dimensional matrices.

Noncommutativity on the moduli space

$$[\hat{x}^{\alpha\dot{\alpha}}, \hat{x}^{\beta\dot{\beta}}] \;=\; \mathrm{i}\theta^{\alpha\dot{\alpha}\beta\dot{\beta}}$$

with: $(\kappa = \pm 1)$

$$\theta^{1\dot{1}2\dot{2}} \;=\; -\theta^{2\dot{2}1\dot{1}} \;=\; -2\mathrm{i}\kappa\varepsilon\theta \quad \text{and} \quad \theta^{1\dot{2}2\dot{1}} \;=\; -\theta^{2\dot{1}1\dot{2}} \;=\; 2\mathrm{i}\varepsilon\theta$$

- representation space: two oscillator Fock space with $|0,0\rangle$

$$\hat{a}_1 \sim \hat{x}^{2\dot{1}} + \hat{x}^{1\dot{2}} \quad \text{and} \quad \hat{a}_2 \sim \hat{x}^{2\dot{2}} - \hat{x}^{1\dot{1}}$$

- derivatives become inner derivations of the above algebra:

$$\frac{\partial}{\partial \hat{x}^{1\dot{1}}} f \sim [\hat{x}^{2\dot{2}}, f]\ , \quad \text{etc.}$$

- integral becomes trace: $\int \mathrm{d}^4 x\, f \;\mapsto\; (2\pi\theta)^2\, \mathrm{tr}_{\,\mathcal{H}} \hat{f}$

Matrix Models from Noncommutativity

Sections ω of the bundle defining supertwistor space are now matrix valued.

Noncommutativity on the twistor space

Induced algebra:

$$
\begin{aligned}
[\hat{\omega}^1_\pm, \hat{\omega}^2_\pm] &= 2(\kappa-1)\varepsilon\lambda_\pm\theta\ , & [\hat{\bar{\omega}}^1_\pm, \hat{\bar{\omega}}^2_\pm] &= -2(\kappa-1)\varepsilon\bar{\lambda}_\pm\theta\ ,\\
[\hat{\omega}^1_+, \hat{\bar{\omega}}^1_+] &= 2(\kappa\varepsilon-\lambda_+\bar{\lambda}_+)\theta\ , & [\hat{\omega}^1_-, \hat{\bar{\omega}}^1_-] &= 2(\kappa\varepsilon\lambda_-\bar{\lambda}_- - 1)\theta\ ,\\
[\hat{\omega}^2_+, \hat{\bar{\omega}}^2_+] &= 2(1-\varepsilon\kappa\lambda_+\bar{\lambda}_+)\theta\ , & [\hat{\omega}^2_-, \hat{\bar{\omega}}^2_-] &= 2(\lambda_-\bar{\lambda}_- - \varepsilon\kappa)\theta\ ,
\end{aligned}
$$

Matrix Models

All operators can be seen as infinite dimensional matrices.

$\Rightarrow$ Matrix models from SDYM and hCS theory

explict equivalence again via field expansion.

Large N limit

N: rank of gauge group, limit $N \to \infty$: all MMs equivalent

D-Brane Interpretation

There is an obvious interpretation of the hCS MM in terms of topological B-branes.

B-Type Topological Branes

- D(-1)-, D1-, D3-, and D5-branes
- stack of N D-branes comes with rank N vector bundle
- effective action: $\mathrm{GL}(N,\mathbb{C})$ holomorphic Chern-Simons theory
- i.e. $F^{0,2}=F^{2,0}=0$ (stability missing: $k^{d+1}\wedge F^{1,1}=\gamma k^d$)

hCS MM: stack of n D1|4-branes wrapping $\mathbb{C}P^{1|4}\hookrightarrow\mathcal{P}^{3|4}$.
expand Higgs-fields $\mathcal{X}_\alpha=\mathcal{X}_\alpha^0+\mathcal{X}_\alpha^i\eta_i+\mathcal{X}_\alpha^{ij}\eta_i\eta_j+\dots$

$$
\begin{aligned}
[\mathcal{X}_1^0,\mathcal{X}_2^0] &= 0\ ,\\
[\mathcal{X}_1^i,\mathcal{X}_2^0]+[\mathcal{X}_1^0,\mathcal{X}_2^i] &= 0\ ,\\
\{\mathcal{X}_1^i,\mathcal{X}_2^j\}-\{\mathcal{X}_1^j,\mathcal{X}_2^i\}+[\mathcal{X}_1^{ij},\mathcal{X}_2^0]+[\mathcal{X}_1^0,\mathcal{X}_2^{ij}] &= 0\ ,
\end{aligned}
$$

bodies $\mathcal{X}_\alpha^0$ can be diagonalized: positions of the D1|4-branes

Fermionic directions are “smeared out” even classically.

D-Brane Interpretation

Physical D-branes: topological D-branes + stability condition.

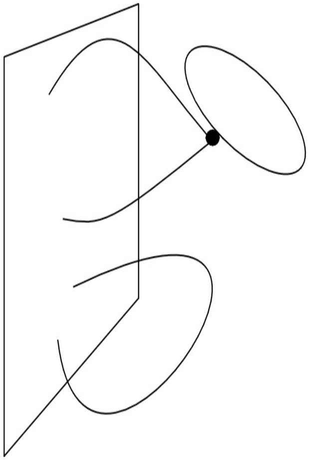
D-Branes in Type IIB String Theory

- D(-1)-, D1-, D3-, ... branes
- stack of N D-branes comes with rank N vector bundle
- effective action: $\mathrm{U}(N)$ SYM reduced from 10 to $p+1$
- curved spaces: $F^{0,2} = F^{2,0} = 0$ and $k^{d+1} \wedge F^{1,1} = \gamma k^d$
- arising Higgs fields: normal fluctuations of D-branes

ADHM Construction and D-Brane Bound States

There is a nice interpretation of the ADHM construction in terms of D-branes.

Bound state of D3-D(-1)-branes (D9-D5-branes + dim. reduction)



Perspective of D3-brane

D3-D3-strings + BPS condition:
SDYM equations
D(-1)-brane: instanton, nontrivial ch_2

Perspective of D(-1)-brane

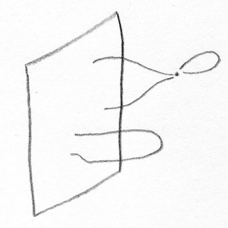
D(-1)-D(-1)-strings:
$\mathcal{N}=(0,1)$ hypmult., adj. $(A_{\alpha\dot\alpha},\chi^i_\alpha)$
D(-1)-D3-strings:
$\mathcal{N}=(0,1)$ hypmult., bifund. $(w_{\dot\alpha},\psi^i)$
D-flatness condition/ADHM eqns.:
$\frac{\mathrm{i}}{16\pi^2}\vec\sigma^{\dot\alpha}{}_{\dot\beta}(\bar w^{\dot\beta}w_{\dot\alpha}+\bar A^{\alpha\dot\beta}A_{\alpha\dot\alpha})=0$

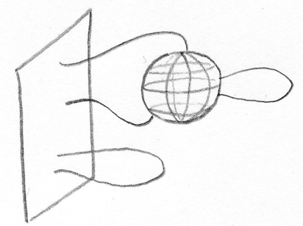
Witten, hep-th/9510135, Douglas, hep-th/9512077,...

ADHM and the SDYM Matrix Model

The SDYM Matrix Model is almost equivalent to the ADHM equations.

- Perspective of D(-1)-branes
- Supersymmetrically extend ADHM eqns.:
  $A_{\alpha\dot{\alpha}} \to A_{\alpha\dot{\alpha}} + \eta^i_{\dot{\alpha}}\chi_{i\alpha}$ and $w_{\dot{\alpha}} \to w_{\dot{\alpha}} + \eta^i_{\dot{\alpha}}\psi_i$
- Drop the D(-1)-D3-strings, i.e. $w_{\dot{\alpha}} \stackrel{!}{=} 0$
- $\Rightarrow$ SDYM MM equations
- How to obtain the full picture?
- Incorporate D(-1)-D3-strings in MM in hCS: D1-D5-strings.





ADHM and the Extended Matrix Models

The hCS MM can be extended to be equivalent to the ADHM equations.

Extended action

$$S_{\text{ext}} = S_{\text{hCS MM}} + \int_{\mathbb{C}P^1_{\text{ch}}} \Omega_{\text{red}} \wedge \operatorname{tr}\left(\beta\bar{\partial}\alpha + \beta\mathcal{A}^{0,1}_{\mathbb{C}P^1}\alpha\right)$$

$\alpha{=}\beta^*$, sections of $\mathcal{O}(1)$, fund. and antifund. of $\mathrm{GL}(N,\mathbb{C})$
(α and β *bosons* not *fermions* as in Witten, hep-th/0312171)

Equations of motion:

$$\bar{\partial}\mathcal{X}_\alpha + [\mathcal{A}^{0,1}_{\mathbb{C}P^1}, \mathcal{X}_\alpha] = 0$$

$$[\mathcal{X}_1, \mathcal{X}_2] + \alpha\beta = 0$$

$$\bar{\partial}\alpha + \mathcal{A}^{0,1}_{\mathbb{C}P^1}\alpha = 0 \quad \text{and} \quad \bar{\partial}\beta + \beta\mathcal{A}^{0,1}_{\mathbb{C}P^1} = 0$$

ADHM and the Extended Matrix Models

Again, the equivalence can be made manifest by a field expansion.

Extended Penrose-Ward transform explicitly

$$\beta = \lambda^{\dot\alpha} w_{\dot\alpha} + \psi^i \eta_i + \gamma \tfrac{1}{2!}\eta_i\eta_j \hat\lambda^{\dot\alpha}\rho^{ij}_{\dot\alpha} + \gamma^2 \tfrac{1}{3!}\eta_i\eta_j\eta_k \hat\lambda^{\dot\alpha}\hat\lambda^{\dot\beta}\sigma^{ijk}_{\dot\alpha\dot\beta} + \dots$$

$$\alpha = \lambda^{\dot\alpha}\varepsilon_{\dot\alpha\dot\beta}\bar w^{\dot\beta} + \dots$$

Truncate the SDYM field content ($\phi^{ij}, \tilde\chi^{ijk}_{\dot\alpha}, G^{ijkl}_{\dot\alpha\dot\beta} = 0$):

- Higher fields in extension also vanish
- This expansion and the hCS MM equations yield the full ADHM-equations.

Conclusions:

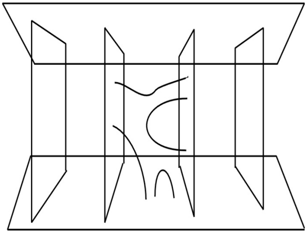
- Extended hCS MM dual to full hCS (as SDYM↔ADHM).
- D(-1)-D3-brane bound states correspond to topological D1-D5-brane systems!

Dimensional Reductions and the Nahm equations

Also for the Nahm-Equations, there is a nice interpretation in terms of D-branes.

Reduction of SDYM eqns. $\mathbb{R}^4 \to \mathbb{R}^3$: Bogomolny monopole eqns.

(static) pair of D3 branes with D1-branes in normal directions



Perspective of D3-brane

static D3-D3-strings + BPS cond.:
Bogomolny equations
(three-dimensional SDYM)
D1-branes: monopoles

Perspective of D1-brane

D1-D1-strings: Nahm equations (one-dimensional SDYM)
D1-D3-strings: Nahm boundary conditions

Diaconescu, hep-th/9608163

Dimensional Reductions and the Nahm equations

For treating the Nahm eqns., one has to change slightly the geometry of twistor space.

Recall

All our MM considerations are based upon
$\mathcal{P}^{3|\cdots} = \mathcal{O}(1) \oplus \mathcal{O}(1) \oplus \ldots \to \mathbb{C}P^1$ and its dim. red. $\mathbb{C}P^{1|4}$.

The twistor space for the Bogomolny equations is $\mathcal{O}(2) \to \mathbb{C}P^1$.

New Calabi-Yau supermanifold

Start from $\mathcal{Q}^{3|4} = \mathcal{O}(2) \oplus \mathcal{O}(0) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1)$
Restrict sections $\hat{\mathcal{Q}}^{3|4}$: $w^1 = y^{\dot{\alpha}\dot{\beta}}\lambda_{\dot{\alpha}}\lambda_{\dot{\beta}}$, $w^2 = y^{\dot{1}\dot{2}}$

Dimensional reductions

$$\hat{\mathcal{Q}}^{3|4} \to \begin{cases} \mathcal{P}^{2|4} := \mathcal{O}(2) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \\ \hat{\mathcal{Q}}^{2|4} := \mathcal{O}(0) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \\ \mathbb{C}P^{1|4} := \mathbb{C}^4 \otimes \Pi\mathcal{O}(1) \end{cases}$$

Dimensional Reductions and the Nahm equations

Different dimensional reductions yield the various field theories in the Nahm construction.

$\hat{Q}^{3|4} = \mathcal{O}(2) \oplus \mathcal{O}(0) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1)|_{\text{res}}$

Upon imposing a reality condition, hCS theory turns into partially hCS theory (→ CR manifolds, etc.): Equiv. to Bogomolny eqns.

Popov, CS, Wolf, JHEP 10 (2005) 058

$\mathcal{P}^{2|4} \ := \ \mathcal{O}(2) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1)$

hCS equations from a holomorphic BF-theory: $\int \Omega \wedge BF^{0,2}$ equivalent to Bogomolny equations

$\hat{Q}^{2|4} \ := \ \mathcal{O}(0) \oplus \mathbb{C}^4 \otimes \Pi\mathcal{O}(1)$

hCS equations from a holomorphic BF-theory: $\int \Omega \wedge BF^{0,2}$ equivalent to Nahm equations

$\mathbb{C}P^{1|4} \ := \ \mathbb{C}^4 \otimes \Pi\mathcal{O}(1)$: again hCS and SDYM matrix models

D-Brane correspondences

We find a list of correspondences between topological and physical D-branes.

Summing up, we have

$$\text{D}5|4\text{-branes in } \mathcal{P}^{3|4} \leftrightarrow \text{D}3|8\text{-branes in } \mathbb{R}^{4|8}$$

$$\text{D}3|4\text{-branes wr. } \mathcal{P}^{2|4} \text{ in } \mathcal{P}^{3|4} \text{ or } \hat{\mathcal{Q}}^{3|4} \leftrightarrow \text{static D}3|8\text{-branes in } \mathbb{R}^{4|8}$$

$$\text{D}3|4\text{-branes wr. } \hat{\mathcal{Q}}^{2|4} \text{ in } \hat{\mathcal{Q}}^{3|4} \leftrightarrow \text{static D}1|8\text{-branes in } \mathbb{R}^{4|8}$$

$$\text{D}1|4\text{-branes in } \mathcal{P}^{3|4} \leftrightarrow \text{D(-}1|8\text{)-branes in } \mathbb{R}^{4|8}$$

straightforward: add diagonal line bundle $\mathcal{D}^{2|4}$, defined by $\omega^1 = \omega^2$

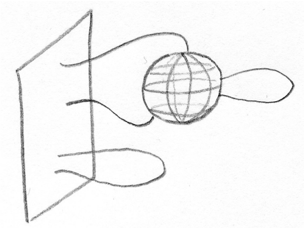
$$\text{D}3|4\text{-branes wrapping } \mathcal{D}^{2|4} \text{ in } \mathcal{P}^{3|4} \leftrightarrow \text{D}1|8\text{-branes in } \mathbb{R}^{4|8} \; .$$

Note:

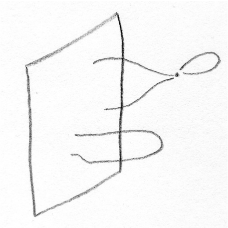
- Branes extend only into chiral fermionic dimensions
- Branes appear in bound state configurations.

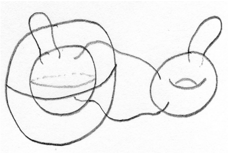
D-brane configuration equivalences

We had topological-physical D-brane equivalences for ADHM and Nahm construction.

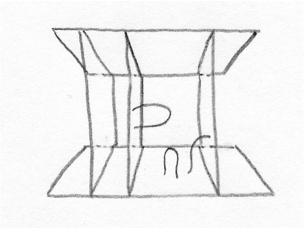


$\Longleftrightarrow$





$\Longleftrightarrow$



But: There are many more.

Done:

- Definition of twistor matrix models
- Extension of the matrix models to
  - full ADHM-equations
  - full Nahm-equations
- Map between topological and physical D-brane bound states

Future Directions:

- Study Nahm equations more closely
- Study mirror configurations?
- Generalize to full Yang-Mills theory
- Carry over results from topological strings to physical ones (e.g. Derived Categories).

Matrix Models and D-Branes in Twistor String Theory

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LMS Durham Symposium 2007

Based on:

- JHEP **0603** (2006) 002, O. Lechtenfeld and CS.