Variations On The Topological B-Model In Twistor String Theory

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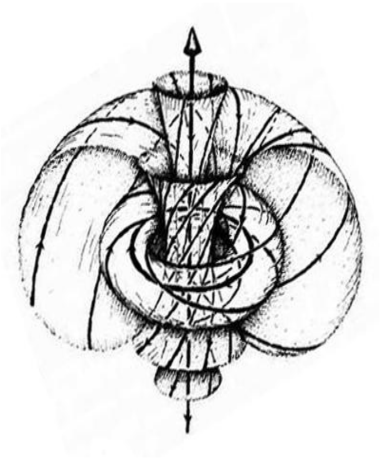
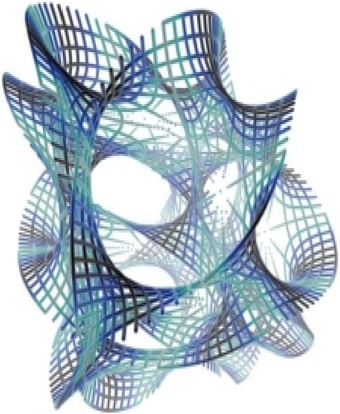
Based on:

- hep-th/0405123 (A.D.Popov, C.S.)
- hep-th/0410292 (C.S.)
- hep-th/0504??? (O.Lechtenfeld, A.D.Popov, C.S.)

Marrying Twistor- and Calabi-Yau Geometry





... with supermanifolds: Witten, hep-th/0312171

Outline

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   $\Rightarrow$ Define a topological B-model on $\mathbb{C}P^{3|4}$.

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$$\Rightarrow \omega^{\alpha} = x^{\alpha\dot{\alpha}}\lambda_{\dot{\alpha}}$$

Moduli $x^{\alpha\dot{\alpha}} \in \mathbb{C}^4$

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Incidence relation $\omega^{\alpha} = x^{\alpha\dot{\alpha}}\lambda_{\dot{\alpha}}$

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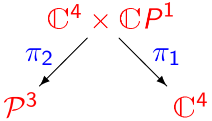

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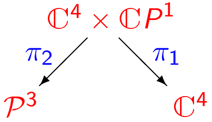

projective line $\mathbb{C}P^1_x$ in $\mathcal{P}^3$ $\leftrightarrow$ point x in $\mathbb{C}^4$

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after introducing a real structure: $\mathcal{P}^3 \cong \mathbb{R}^4 \times \mathbb{C}P^1 \rightarrow \mathbb{R}^4$

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Chern number of $\mathcal{P}^{3|4}$: $T\mathbb{C}P^1$ 2, $\mathcal{O}(1)$ 1, $\Pi\mathcal{O}(1)$ -1,
in total: $c_1 = 0$.
Therefore, there exists a holomorphic measure $\Omega^{3,0|4,0}$.

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Solutions to the $\mathcal{N} = 4$ SDYM equations on $\mathbb{C}^{4|8}$:

$$f_{\dot{\alpha}\dot{\beta}} = 0 \ , \quad \nabla_{\alpha\dot{\alpha}}\chi^{\alpha i} = 0 \ ,$$

$$\Box\phi^{ij} + 2\{\chi^{\alpha i}, \chi^{j}_{\alpha}\} = 0 \ , \quad \nabla_{\alpha\dot{\alpha}}\tilde{\chi}^{\dot{\alpha}ijk} - [\chi^{[i}_{\alpha}, \phi^{jk]}] = 0$$

$$\varepsilon^{\dot{\alpha}\dot{\gamma}}\nabla_{\alpha\dot{\alpha}}G^{[ijkl]}_{\dot{\gamma}\dot{\delta}} + \{\chi^{[i}_{\alpha}, \tilde{\chi}^{jkl]}_{\dot{\delta}}\} - [\phi^{[ij}, \nabla_{\alpha\dot{\delta}}\phi^{kl]}] = 0$$

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- $y_1 = \eta_1\eta_2,\; y_2 = \eta_3\eta_4$:

  $S_{\mathcal{P}^{3\oplus 2|0}} = \int \mathrm{d}^4x \,\mathrm{tr}\, \left(G^{\dot\alpha\dot\beta} f_{\dot\alpha\dot\beta} - \phi^{(1}\Box\phi^{2)} \right)$

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Advantages:

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- More manifolds for testing mirror symmetry

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Fattened complex manifolds fit nicely in the picture!

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Resulting action:

$$S_{\text{red}} := \int_{\mathbb{C}P^{1|4}_{\text{ch}}} \mathrm{d}\lambda_\pm \wedge \mathrm{d}\bar{\lambda}_\pm \; H^\pm \, \mathrm{tr} \left(\varepsilon^{\alpha\beta} \mathcal{X}^\pm_\alpha \bar{\partial}_{\bar{\lambda}_\pm} \mathcal{X}^\pm_\beta + 2\mathcal{A}^\pm \left[\mathcal{X}^\pm_1, \mathcal{X}^\pm_2 \right] \right)$$

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$\Rightarrow$ *Construct Matrix Model solutions by twistor techniques*

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Alternative approach:

Use diffeomorphism $\mathcal{P}^{3|4} \cong \mathbb{R}^{4|8} \times \mathbb{C}P^1$ and make bosonic moduli noncommutative:

$$[\hat{x}^{1\dot{1}}, \hat{x}^{2\dot{2}}] = 2\theta\ , \quad [\hat{x}^{1\dot{2}}, \hat{x}^{2\dot{1}}] = -2\theta$$

Derivatives and fields become operators/matrices in a Fock space.

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In the topological B-model there are

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Interesting, as similar instantons are used
to complete the SDYM picture to full SYM theory
for handling amplitudes.

Variation III: Monopoles

Idea: SDYM on $\mathbb{R}^4$ $\longrightarrow$ Bogomolny eqn. on $\mathbb{R}^3$

$$F_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma} \longrightarrow F_{ab} = \varepsilon_{abc}D_c\phi$$

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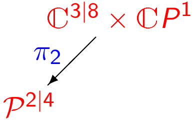

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(More natural description in terms of Cauchy-Riemann manifolds.)

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  - Further variations like monopoles lead to interesting new structures (CR manifolds) and their applications.
- Super Calabi-Yau geometry should be studied in more detail.

Variations On The Topological B-Model In Twistor String Theory

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Bad Honnef 2005