Looking For the Classical (2,0)-Theory

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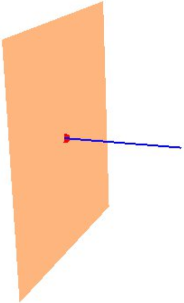
Based on work w. S Palmer, G Demessie, B Jurčo, M Wolf, P Ritter, L Schmidt:

- Higher Gauge Theory: 1203.5757, 1308.2622, 1311.1977, 1406.5342, 1512.07554, 1602.03441, 1604.?????
- Integrability: 1105.3904, 1205.3108, 1305.4870, 1312.5644, 1403.7185

Motivation: The Dynamics of Multiple M5-Branes

To understand M-theory, an effective description of M5-branes would be very useful.

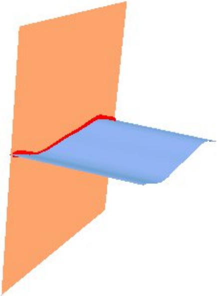


D-branes

- D-branes interact via strings.
- Effective description: theory of endpoints
- Parallel transport of these: Gauge theory
- Study string theory via gauge theory

M5-branes

- M5-branes interact via M2-branes.
- Eff. description: theory of self-dual strings
- Parallel transport: Higher gauge theory
- Can we study M-theory via HGT?



What we know

Multiple M5-branes are described by a $\mathcal{N} = (2, 0)$ superconformal field theory.

What we know:

- String theory considerations: conformal fixed point in 6d
  Witten, Strominger 1995
- Field content: $\mathcal{N} = (2, 0)$ supermultiplet in 6d:
  - a self-dual 3-form field strength
  - five (Goldstone) scalars
  - fermionic partners
- A theory of essentially tensionless light strings
- Supergravity decouples, so study string dynamics separately
- Observables: Wilson surfaces, i.e. parallel transport of strings
- No Lagrangian description known
- As important as $\mathcal{N} = 4$ super Yang-Mills for string theory
- Huge interest in string theory: AGT, AdS$_7$-CFT$_6$, S-duality, ...
- Mathematics: Geom. Langlands, Khovanov Homology, ...

Parallel Transport of Strings is Problematic

The lack of surface ordering renders a parallel transport of strings problematic.

Parallel transport of particles in representation of gauge group G:

- holonomy functor hol : path $\gamma \mapsto \mathsf{hol}(\gamma) \in \mathsf{G}$
- $\mathsf{hol}(\gamma) = P\exp(\int_\gamma A)$, P: path ordering, trivial for $\mathsf{U}(1)$.

Parallel transport of strings with gauge group $\mathsf{U}(1)$:

- map hol : surface $\sigma \mapsto \mathsf{hol}(\sigma) \in \mathsf{U}(1)$
- $\mathsf{hol}(\sigma) = \exp(\int_\sigma B)$, B: connective structure on gerbe.

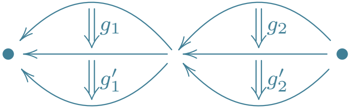
Nonabelian case:

- much more involved!
- no straightforward definition of surface ordering

Naïve no-go theorem

Naively, there is no non-abelian parallel transport of strings.

Imagine parallel transport of string with gauge degrees in Lie(G):



Consistency of parallel transport requires:

$$(g_1' g_2')(g_1 g_2) = (g_1' g_1)(g_2' g_2)$$

This renders group G abelian.

Eckmann and Hilton, 1962
Physicists 80'ies and 90'ies

Way out: 2-categories, Higher Gauge Theory.

Two operations $\circ$ and $\otimes$ satisfying Interchange Law:

$$(g_1' \otimes g_2') \circ (g_1 \otimes g_2) = (g_1' \circ g_1) \otimes (g_2' \circ g_2) \ .$$

Objection to a classical (2,0)-theory

Without coupling constant, there shouldn't be classical effective descriptions in M-theory.

Standard objection beyond the previous no-go theorem:

- theory at conformal fixed points $\Rightarrow$ no dimensionful parameter
- fixed points are isolated $\Rightarrow$ no dimensionless parameter
- "No parameters $\Rightarrow$ no classical limit $\Rightarrow$ no Lagrangian."

Answers:

- Same arguments for M2-brane Schwarz, 2004
- There, integer parameters arose from orbifold $\mathbb{R}^8/\mathbb{Z}_k$
- Same should happen for M5-branes
- Even if no Lagrangian, BPS-states may exist classically
- Even if not, study quantum features of related theories.

4-Slide Crash Course on Higher Gauge Theory

Fastest Route to Higher Gauge Theory

One easily constructs local higher gauge theory using NQ-manifolds.

N-manifolds, NQ-manifold

- $\mathbb{N}$-graded manifold with coordinates of degree $0, 1, 2, \dots$

$$\begin{tikzcd} M_0 & M_1 \arrow[l] & M_2 \arrow[l] & \dots \arrow[l] \\ \text{manifold} \arrow[u] & & \text{linear spaces} \arrow[ul] \arrow[u] & \end{tikzcd}$$

- Morphisms $\phi : M \to N$ are maps $\phi^* : \mathcal{C}^\infty(N) \to \mathcal{C}^\infty(M)$
- NQ-manifold: vector field Q of degree 1, $Q^2 = 0$
- Physicists: think ghost numbers, BRST charge, SFT

Examples:

- Tangent algebroid $T[1]M$, $\mathcal{C}^\infty(T[1]M) \cong \Omega^\bullet(M)$, $Q = \mathrm{d}$
- Lie algebra $\mathfrak{g}[1]$, coordinates ξ^a of degree 1:

$$Q = -\tfrac{1}{2} f_{ab}^c \xi^a \xi^b \frac{\partial}{\partial \xi^c}$$

Condition $Q^2 = 0$ is equivalent to Jacobi identity for f_{ab}^c

L_∞-Algebras, Lie 2-Algebras

NQ-manifolds provide an easy definition of L_∞-algebras.

Lie n-algebroid or n-term L_∞-algebroid:

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Lie n-algebra, n-term L_∞-algebra or Lie n-algebra:

$$* \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Example: Lie 2-algebra

- NQ-manifold: $* \leftarrow W[1] \leftarrow V[2] \leftarrow * \leftarrow \ldots$, coords. w^a, v^i
- Homological vector field:

$$Q = -m_i^a v^i \frac{\partial}{\partial w^a} - \tfrac{1}{2} m_{ab}^c w^a w^b \frac{\partial}{\partial w^c} - m_{ai}^j w^a v^i \frac{\partial}{\partial v^j} - \tfrac{1}{3!} m_{abc}^i w^a w^b w^c \frac{\partial}{\partial v^i}$$

- Structure constants: higher products μ_i on $W \leftarrow V[1]$

$$\mu_1(\tau_i) = m_i^a \tau_a \ , \quad \mu_2(\tau_a, \tau_b) = m_{ab}^c \tau_c \ , \quad \ldots \ , \quad \mu_3(\tau_a, \tau_b, \tau_c) = m_{abc}^i \tau_i$$

- $Q^2 = 0$: Higher or homotopy Jacobi identity, e.g.

$$\mu_2(w_1, \mu_2(w_2, w_3)) + \text{cycl.} = \mu_1(\mu_3(w_1, w_2, w_3))$$

Local Higher Gauge Theory via NQ-Manifolds

One easily constructs local higher gauge theory using NQ-manifolds.

Ordinary gauge theory:

- Gauge potential from morphism of N-manifolds:

$$a : T[1]M \to \mathfrak{g}[1] \quad \longrightarrow \quad A^a_\mu \mathrm{d}x^\mu := a^*(\xi^a)$$

- Curvature: failure of a to be morphism of NQ-manifold:

$$F^a := (\mathrm{d} \circ a^* - a^* \circ Q)(\xi^a) = \mathrm{d}A^a + \tfrac{1}{2} f^a_{bc} A^b \wedge A^c$$

- Gauge transformations: flat homotopies between morphisms.

Higher gauge theory:

- Gauge potentials $T[1]M \to (W[1] \leftarrow V[2])$:

$$A^a_\mu \mathrm{d}x^\mu := a^*(w^a) \quad \text{and} \quad B^i_{\mu\nu} \mathrm{d}x^\mu \wedge \mathrm{d}x^\nu = a^*(v^i)$$

- Curvature: failure of a to be morphism of NQ-manifold:

$$\mathcal{F} := \mathrm{d}A + \tfrac{1}{2}\mu_2(A, A) + \mu_1(B)$$
$$\mathcal{H} := \mathrm{d}B + \mu_2(A, B) + \tfrac{1}{3!}\mu_3(A, A, A)$$

Local Higher Gauge Theories

The most interesting higher gauge theories for us live in 6 and 4 dimensions.

- "Fake curvature": $\mathcal{F} = \mathrm{d}A + \frac{1}{2}\mu_2(A, A) - \mu_1(B) = 0$
  Vanishing makes parallel transport reparam. invariant.
- 3-form curvature: $\mathcal{H} = \mathrm{d}B + \mu_2(A, B) + \frac{1}{3!}\mu_3(A, A, A)$

Gauge part of (2,0) theory

If (2,0) theory on $\mathbb{R}^{1,5}$ is a higher gauge theory, then gauge part is:

$$\mathcal{H} = *\mathcal{H}\ , \quad \mathcal{F} = 0\ .$$

Non-Abelian Self-Dual Strings

BPS equation for (2,0) theory on $\mathbb{R}^4$ ($\sim$ monopoles in 4d SYM)

$$\mathcal{H} = *\left(\mathrm{d}\Phi + \mu_2(A, \Phi)\right)\ , \quad \mathcal{F} = 0\ .$$

The Global Picture: Categorification

We will need to use some very simple notions of category theory, an esoteric subject noted for its difficulty and irrelevance.
G. Moore and N. Seiberg, 1989

What does categorification mean?
One of Jeff Harvey's questions to identify the "generation PhD>1999" at Strings 2013.

Categorification

Categorification provides some guidelines in the construction of higher objects.

Category theory: excellent tool for deformations/generalizations.

Notions used: categorification, internalization and enrichment.

Idea: Mathematical objects are stuff, structures, structure eqns.

Translate as follows:

- stuff (sets) becomes categories
- structures (functions) become functors
- structure equations become structure isomorphisms

Example: 2-Groups

Categorifying a group, we arrive at the notion of a 2-group.

Group:

- Stuff: Underlying set G, unit $\mathbb{1}$
- Structure: Multiplication, inverse
- Structure equations: associativity, $g^{-1}g = \mathbb{1}$, $\mathbb{1}g = g\mathbb{1} = g$

2-Group:

- Stuff: A category $\mathscr{C}$, unit object $\mathbb{1}$
- Structure: Multiplication bifunctor $\otimes$, inverse functor inv
- Structure isomorphisms:
  - $\mathsf{a}_{x,y,z} : (x \otimes y) \otimes z \Rightarrow x \otimes (y \otimes z)$
  - $\mathsf{l}_x : x \otimes \mathbb{1} \Rightarrow x$, $\mathsf{r}_x : \mathbb{1} \otimes x \Rightarrow x$
  - $\mathsf{inv}(x) \otimes x \Rightarrow \mathbb{1} \Leftarrow x \otimes \mathsf{inv}(x)$

Example: Strict 2-Group $\mathsf{G} \ltimes \mathsf{H} \rightrightarrows \mathsf{G}$,

- a, l, r all trivial, $\mathsf{inv}(x) \otimes x = \mathbb{1} = x \otimes \mathsf{inv}(x)$
- $\mathrm{id}(g) = (g, \mathbb{1}_{\mathsf{H}})$, $(g_1, h_1) \otimes (g_2, h_2) = (g_1 g_2, h_1(g_1 \rhd h_2))$, etc.

Categorical Description of Principal Bundles

Descent data for principal bundles is encoded in a functor.

The cover $\sqcup_a U_a$ of a manifold M encoded in the Čech groupoid:

$$\check{\mathscr{C}}(U) \ : \quad \bigsqcup_{a,b} U_{ab} \rightrightarrows \bigsqcup_{a} U_a \ , \quad U_{ab} \circ U_{bc} = U_{ac} \ .$$

Principal G-bundle

Transition functions are nothing but a functor $g : \check{\mathscr{C}}(U) \to (\mathsf{G} \rightrightarrows *)$

$$\begin{tikzcd}
\sqcup U_{ab} \arrow[r, "g_{ab}"] \arrow[d, shift left] \arrow[d, shift right] & \mathsf{G} \arrow[d, shift left] \arrow[d, shift right] \\
\sqcup U_a \arrow[r, "*"] & *
\end{tikzcd}$$

$$g_{ab} g_{bc} = g_{ac}$$

Equivalence relations: natural isomorphisms.

Use higher categories: Higher bundles including gerbes

Connections and Finite Gauge Transformations

Higher connections and finite gauge transformations are readily derived.

First option: Integrate infinitesimal description from above

- Procedure rather involved
- Result usually only categorically equivalent to expected one

Second option: Differentiate gauge 2-group Ševera

- Functors from manifolds to transition functions of principal G-bundles over $M \times \mathbb{R}^{0|1} \to M$
- Moduli space is N-manifold $\mathsf{Lie}(\mathsf{G})[1]$
- Induced action of $\mathsf{Hom}(\mathbb{R}^{0|1}, \mathbb{R}^{0|1})$ on moduli space yields Q.
- Directly generalizes to L_∞- and quasi-groupoids

Finite gauge transformations: B Jurčo, CS & M Wolf

- Natural transformations yield Maurer-Cartan forms etc.
- Readily read off gauge transformations

Summary of our construction

Input:

- some higher Lie groupoid as higher space-time
- some higher Lie groupoid as higher gauge group

Output:

- higher principal bundle
- higher curvatures
- finite gauge transformations

Context:

Many Suggested Models are Higher Gauge Theories

M2-Brane Models as Higher Gauge Theories

The ABJM model can be completed to a higher gauge theory.

- Most dualities in string theory between Yang-Mills theories.
- And in M-theory?
  M2-branes: Chern-Simons-matter theories
  M5-branes: Tensor-multiplet theories
- These can be put on equal footing. S Palmer&CS, 1311.1997
- Note: The ABJM gauge structures form strict Lie 2-algebras.
- Here: need Lie 3-algebra:

$$\begin{pmatrix} 0 & \mathfrak{gl}(N,\mathbb{C}) \\ 0 & 0 \end{pmatrix} \xrightarrow{\mu_1} \begin{pmatrix} \mathfrak{u}(N) & \mathfrak{gl}(N,\mathbb{C}) \\ 0 & \mathfrak{u}(N) \end{pmatrix} \xrightarrow{\mu_1} \begin{pmatrix} \mathfrak{u}(N) & 0 \\ 0 & \mathfrak{u}(N) \end{pmatrix}$$

- Action implementing fake curvature conditions:

$$S_{\mathrm{ABJM}} = \int_{\mathbb{R}^{1,2}} \mathrm{tr}\left(\tfrac{k}{4\pi}\eta\ A \wedge (\mathrm{d}A + \tfrac{1}{3}[A,A]) \right.$$
$$\left. - \nabla Z_A^\dagger \wedge *\nabla Z^A - *\mathrm{i}\bar{\psi}^A \wedge \not{\nabla} \psi_A \right) + V$$

$$S_{\mathrm{HGT}} = S_{\mathrm{ABJM}} + \int_{\mathbb{R}^{1,2}} \mathrm{tr}\left(\lambda_1^\dagger \wedge (F - \mu_1(B)) \right.$$
$$\left. + \lambda_2^\dagger (H - \mu_1(C)) + \lambda_3^\dagger \mu_1(\lambda_2) \right)$$

Tensor Hierarchy Models are Higher Gauge Theories

There is much more evidence for using higher structures in M-theory.

Proposal for 6d (1,0) models from tensor hierarchies

Samtleben et al., 1108.4060, also 1108.5131

Gauge structure:

- Graded vector space: $\mathfrak{g} \xleftarrow{\mathsf{h}} \mathfrak{h} \xleftarrow{\mathsf{g}} \mathfrak{l}$
- Maps with structure relations:

$$\begin{aligned} \mathsf{h}(\mathsf{g}(\lambda)) &= 0 \\ \mathsf{f}(\mathsf{h}(\chi),\gamma) - \mathsf{h}(\mathsf{d}(\mathsf{h}(\chi),\gamma)) &= 0 \\ \mathsf{f}(\gamma_{[1},\mathsf{f}(\gamma_2,\gamma_{3]})) - \tfrac{1}{3}\mathsf{h}(\mathsf{d}(\mathsf{f}(\gamma_{[1},\gamma_2),\gamma_{3]})) &= 0 \\ \mathsf{g}(\mathsf{b}(\chi_1,\mathsf{h}(\chi_2))) - 2\mathsf{d}(\mathsf{h}(\chi_1),\mathsf{h}(\chi_2)) &= 0 \\ \mathsf{g}(\mathsf{f}^*(\gamma,\lambda) - \mathsf{d}^*(\mathsf{h}^*(\lambda),\gamma) + \mathsf{b}(\mathsf{g}(\lambda),\gamma)) &= 0 \end{aligned}$$

- Relations for a symplectic NQ-manifold in degrees $1, 2, 3$
- This is a higher gauge theory without fake curvature condition
- Overlap with models by Chu and Ho & Matsuo.

How to construct (2,0)-theories systematically

Twistor Description of Higher Gauge Fields

Using twistor spaces, one can map holomorphic data to solutions to field equations.

Recall the principle of the Penrose-Ward transform:

- We construct a double fibration

$$\begin{tikzcd} & F \arrow[dl] \arrow[dr] & \\ P & & M \end{tikzcd}$$

  P: twistor space, F: correspondence space
- $H^n(P,\mathfrak{S})$ (e.g. vector bundles) $\xleftrightarrow{1:1}$ sols. to field equations.
- Works for
  - instantons, monopoles, ...
  - solutions to $\mathcal{N}=4$ super Yang-Mills theory
- Idea: use this to construct 6d (2,0) superconformal eoms

Known Examples of Twistor Descriptions

For Yang-Mills theories and its BPS subsectors, there is a wealth of twistor descriptions.

$$\begin{tikzcd}
& \mathbb{C}^4\times\mathbb{C}P^1 \arrow[dl] \arrow[dr] & \\
\mathbb{C}P^3_\circ & & \mathbb{C}^4
\end{tikzcd}$$

Instantons
hol. vector bundle

$$\begin{tikzcd}
& \mathbb{C}^3\times\mathbb{C}P^1 \arrow[dl] \arrow[dr] & \\
T\mathbb{C}P^1 & & \mathbb{C}^3
\end{tikzcd}$$

Monopoles
hol. vector bundle

$$\begin{tikzcd}
& \mathbb{C}^{4|12}\times\mathbb{C}P^1\times\mathbb{C}P^1 \arrow[dl] \arrow[dr] & \\
P^{5|6} & & \mathbb{C}^{4|12}
\end{tikzcd}$$

Super Yang-Mills
hol. vector bundle

$$\begin{tikzcd}
& \mathbb{C}^6\times\mathbb{C}P^3 \arrow[dl] \arrow[dr] & \\
P^6 & & \mathbb{C}^6
\end{tikzcd}$$

abelian $\mathcal{H}=*\mathcal{H}$
hol. gerbe

Hughston, Murray, Eastwood, CS & M Wolf, Mason et al.

Note: last twistor space reduces nicely to the above ones.

Constructing (2,0)-theories

Penrose-Ward transform for non-abelian self-dual tensor multiplet.

$$\begin{tikzcd}
& \mathbb{C}^{6|16} \times \mathbb{C}P^3 \arrow[dl] \arrow[dr] & \\
P^{6|4} & & \mathbb{C}^{6|16}
\end{tikzcd}$$

non-abelian self-dual tensor multiplet
hol. principal 2-bundle
also: semistrict, simplicial, 3-bundles, ...

B Jurčo, CS & M Wolf

Note:

- $P^{6|4}$ is a straightforward SUSY generalization of P^6
- EOMs, abelian: $\mathcal{H} = \star\mathcal{H}$, $\mathcal{F} = 0$, $\not{\nabla}\psi = 0$, $\Box\phi = 0$
- $\mathcal{N} = (2,0)$ SC non-abelian tensor multiplet EOMs!
- Reduces search for (2,0)-theory to search for gauge structure
- Similarly: Twistor description of self-dual strings.

You may be talking about the empty set,
I want to see some examples...

A Categorified Nahm Transform

Many ingredients are still missing, more work needs to be done.

- Usually, we wouldn't use twistors to construct solutions
- Instead: ADHM- and AHDMN-constructions
- Equations explicitly solvable, moduli space under control, etc.
- No higher Nahm transform so far
- Substantially more ingredients than twistor construction:
  - associated 2-vector bundle
  - Stringor bundle (categorified Spinor bundle)
  - categorified Dirac operator
- Solutions to these problems: Major mathematical progress
- More work is needed...

Review: The 't Hooft-Polyakov Monopole

The 't Hooft-Polyakov Monopole is a non-singular solution with charge 1.

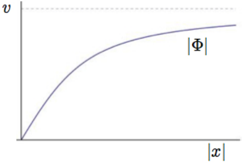
Recall 't Hooft-Polyakov monopole (e_i generate $\mathfrak{su}(2)$, $\xi = v|x|$):

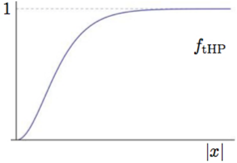
$$\Phi = \frac{e_i x^i}{|x|^2}(\xi \coth(\xi) - 1)~, \quad A = \varepsilon_{ijk}\frac{e_i x^j}{|x|^2}\left(1 - \frac{\xi}{\sinh(\xi)}\right)~\mathrm{d}x^k$$

- At S^2_∞: $\Phi \sim g(\theta) e_3 g(\theta)^{-1}$.
  $g(\theta) : S^2_\infty \rightarrow \mathsf{SU}(2)/\mathsf{U}(1)$: winding 1
- Charge $q = 1$ with

$$2\pi q = \tfrac{1}{2}\int_{S^2_\infty} \frac{\mathrm{tr}\,(F^\dagger \Phi)}{||\Phi||} \quad \text{with} \quad ||\Phi|| := \sqrt{\tfrac{1}{2}\,\mathrm{tr}\,(\Phi^\dagger \Phi)}$$

- Higgs field non-singular:





Elementary Solutions: A Non-Abelian Self-Dual String

We can write down a non-abelian self-dual string with winding number 1.

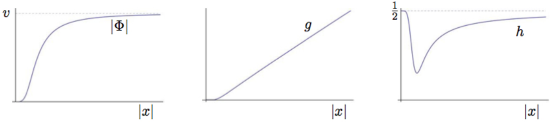
Self-Dual String (Lie 2-algebra $\mathfrak{su}(2)\times\mathfrak{su}(2)\xleftarrow{\mu_1}\mathbb{R}^4$, $\xi=v|x|^2$):

$$\Phi=\frac{e_\mu x^\mu}{|x|^3}\,f(\xi)\;,\quad B_{\mu\nu}=\varepsilon_{\mu\nu\kappa\lambda}\frac{e_\kappa x^\lambda}{|x|^3}\,g(\xi)\;,\quad A_\mu=\varepsilon_{\mu\nu\kappa\lambda}D(e_\nu,e_\kappa)\,\frac{x^\lambda}{|x|^2}\,h(\xi)$$

- Solves indeed $H=\star\nabla\Phi$ for right $f(\xi)$, $g(\xi)$, $h(\xi)$
- At S^∞_3: $\Phi\sim g(\theta)\triangleright e_4$. $g(\theta):S^3_\infty\to\mathsf{SU}(2)$ has winding 1.
- Charge $q=1$:

$$(2\pi)^3q=\tfrac{1}{2}\int_{S^3_\infty}\frac{(H,\Phi)}{||\Phi||}\quad\text{with}\quad ||\Phi||:=\sqrt{\tfrac{1}{2}(\Phi,\Phi)}\;,$$

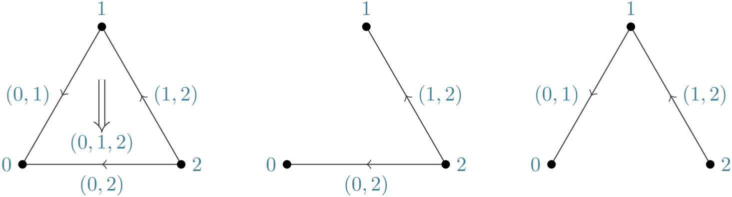
- Higgs field non-singular:



Extensions: Quasi-Categories

Most general, currently available approach: quasi-categories

- Quasi- or ∞-categories: models for $(\infty, 1)$-categories.
  Boardman, Vogt, Joyal
- Example: the nerve of a category
- Simplicial sets, which are Kan complexes: Horns can be filled:



- Functors: simplicial maps
- Natural transformations: simplicial homotopies
- Ševera's differentiation: yields corresponding L_∞-algebra.
- Higher gauge theory: 1604.????, B Jurčo, CS and M. Wolf

The String Group

A very interesting case: The string group.

- Monopole/instanton solutions: gauge group from spin group
  $\mathsf{Spin}(3) \cong \mathsf{SU}(2)$, $\mathsf{Spin}(4) \cong \mathsf{SU}(2) \times \mathsf{SU}(2)$
- Higher analogue of the spin group: String group
  Stolz, Teichner, Witten, ...
- Def. via Whitehead tower (iteratively delete homotopy groups)

$$\ldots \rightarrow \mathsf{String}(n) \rightarrow \mathsf{Spin}(n) \rightarrow \mathsf{Spin}(n) \rightarrow \mathsf{SO}(n) \rightarrow \mathsf{O}(n)$$

- Definition only up to homotopy, as a group: ∞-dimensional
- 2-group models:
  - ∞-dimensional strict 2-group Baez et al., Nikolaus et al.
  - finite-dimensional quasi 2-group Schommer-Pries
- Higher gauge theory 1602.03441, G A Demessie and CS
- Conjecture: Gauge 2-group for M5-branes is $\mathsf{String}(E_8)$

Generalized Higher Gauge Theory

Replacing spacetime by a categorified space yields interesting features.

Return to picture from beginning:

Gauge theory	$a : T[1]M \to \text{Lie algebra}[1]$
Higher gauge theory	$a : T[1]M \to L_\infty\text{-algebra}[1]$

Generalize this:

Generalized higher gauge theory	$a : T^*[2]T[1]M \to L_\infty\text{-algebra}[1]$

Generalized higher gauge theory: P Ritter, CS and L Schmidt

- $T[1]M$ replaced by exact Courant algebroid $T^*M \oplus TM$
- relation to generalized geometry/double field theory
- M is replaced by a categorified space $T^*M \rightrightarrows M$
- Gauge connection contains a cov. constant vector field
- Lambert & Papageorgakis suggested eoms for
  3-Lie algebra valued tensor multiplet
- Naturally interpreted in generalized higher gauge theory

Conclusions

Summary and Outlook.

Summary:

- ✓ Clear physical and mathematical motivation to study HGT
- ✓ Many suggested models are HGT or GHGT
- ✓ Explicit higher monopole and instanton solutions
- ✓ Various twistor constructions with principal 2-bundles
- ✓ 6d superconformal tensor multiplet equations
- ✓ Higher groupoid gauge theories on higher groupoids

Future directions:

- ▷ Study more general higher groups
- ▷ Extend and study twistor constructions
- ▷ Find more solutions
- ▷ Continue translation of higher ADHM-constructions

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