Towards an M5-brane model: A 6d superconformal field theory

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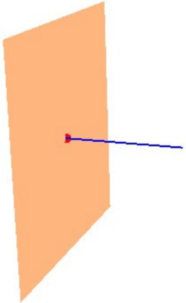
Based on:

- CS & L Schmidt, arXiv:1705.02353
- CS & L Schmidt, arXiv:1712.06623

Motivation: Dynamics of multiple M5-branes

To understand M-theory, an effective description of M5-branes would be very useful.

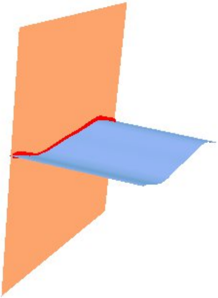


D-branes

- D-branes interact via strings.
- Effective description: theory of endpoints
- Parallel transport of these: Gauge theory
- Study string theory via gauge theory

M5-branes

- M5-branes interact via M2-branes.
- Eff. description: theory of self-dual strings
- Parallel transport: Higher gauge theory
- Long sought $(2,0)$-theory a HGT?



Outline

- The (2,0)-Theory: What we know and what we want
- Arguments against existence of classical M5-brane model
- Higher Gauge Theory: Lightening review
- Guidance from BPS self-dual strings
- The 6d superconformal field theory
- Consistency checks
- Problems and potential solutions

The (2,0)-Theory: What we know and want

What we know about the (2,0)-theory

Pre-history:

- Conformal QFTs: particularly interesting and important
- Conformal algebra on $\mathbb{R}^{p,q}$: $\mathfrak{so}(p+1,q+1)$
- Supersymmetric extensions only for $p+q\leq 6$ Nahm, 1978
- Examples for $p+q\leq 4$ known for long time
- Belief: $p+q=4$ maximum for interacting QFTs

String theory: Witten, 1995

- Type IIB superstring theory on $\mathbb{R}^{1,5}\times K_3$
- Moduli space has orbifold singularities of ADE-type
- At singularities: volume of $S^2 \hookrightarrow K_3$ vanishes
- D3-branes wrapping $S^2 \hookrightarrow K_3$ become massless strings
- B-field self-dual: self-dual strings, SUGRA decouples
- $\Rightarrow$ (2,0)-theory, a six-dimensional $\mathcal{N}=(2,0)$ SCFT

More on the (2,0)-theory

- Also appears in M-theory — Witten, Strominger 1995/1996
  - self-dual strings: boundaries of M2- between M5-branes
  - become massless, if M5-branes approach each other
  - description of stacks of parallel M5-branes
- Field content: $\mathcal{N}=(2,0)$ tensor multiplet — Nahm 1978
  - a self-dual 3-form field strength
  - five (Goldstone) scalars
  - fermionic partners
- Observables: Wilson surfaces, i.e. parallel transport of strings
- Belief: No Lagrangian description
- As important as $\mathcal{N}=4$ super Yang-Mills for string theory
- Huge interest in string theory: AGT, AdS_7-CFT_6, S-duality, ...
- Mathematics: Geom. Langlands, Khovanov Homology, ...

A successful M5-brane model should have the following properties:

- Contain an interacting, self-dual 2-form gauge potential
- Based on a sound mathematical foundation: higher bundles
- Field content of the $(2,0)$-theory, $\mathcal{N}=(1,0)$ supersymmetric
- Gauge structure natural, match some expectations (ADE, ...)
- Non-trivial coupling, interacting field theory
- Possible restriction to free $\mathcal{N}=(2,0)$ tensor multiplet
- contains the non-abelian self-dual string soliton as BPS state
- Reduction to 4d SYM theory with ADE gauge algebras
- and to 3d Chern–Simons-matter models with discrete coupling
- match expected moduli space of $(2,0)$-theory

Arguments against existence of classical M5-brane model

Non-abelian parallel transport of strings problematic:

$$\begin{tikzcd}[column sep=huge]
\bullet & {} \arrow[l, bend right=60, ""{name=U1, below}] \arrow[l, ""{name=M1}] \arrow[l, bend left=60, ""{name=D1, above}] & \bullet \arrow[l, bend right=60, ""{name=U2, below}] \arrow[l, ""{name=M2}] \arrow[l, bend left=60, ""{name=D2, above}]
\arrow[from=U1, to=M1, Rightarrow, "g_1"] \arrow[from=M1, to=D1, Rightarrow, "g_1'"] \arrow[from=U2, to=M2, Rightarrow, "g_2"] \arrow[from=M2, to=D2, Rightarrow, "g_2'"]
\end{tikzcd}$$

Consistency of parallel transport requires:

$$(g_1' g_2')(g_1 g_2) = (g_1' g_1)(g_2' g_2)$$

This renders group G abelian.

Eckmann and Hilton, 1962
Physicists 80'ies and 90'ies

Way out: 2-categories, Higher Gauge Theory.

Two operations $\circ$ and $\otimes$ satisfying Interchange Law:

$$(g_1' \otimes g_2') \circ (g_1 \otimes g_2) = (g_1' \circ g_1) \otimes (g_2' \circ g_2) \ .$$

Standard objection beyond the previous no-go theorem:

- theory at conformal fixed points $\Rightarrow$ no dimensionful parameter
- fixed points are isolated $\Rightarrow$ no dimensionless parameter
- "No parameters $\Rightarrow$ no classical limit $\Rightarrow$ no Lagrangian."
  string theory folklore
- Furthermore: no continuous deformations of free theory
  Bekaert, Henneaux, Sevrin (1999)

Answers:

- Same arguments for M2-brane Schwarz, 2004
- There, integer parameters arose from orbifold $\mathbb{R}^8/\mathbb{Z}_k$
- Same should happen for M5-branes

Final common objection: Dimensional reduction is unclear.

- (2,0)-theory should reduce to $\mathcal{N}=2$ SYM theory in 5d
- Reduction on $\mathbb{R}^{1,4}\times S^1$, radius R yields volume form $2\pi R\,\mathrm{d}^5x$
- Conformal invariance of $F\wedge *F$ requires volume form $\frac{1}{R}\mathrm{d}^5x$

Our solution:

- Reduction to $\mathcal{N}=2$ SYM in 4d works fine
- Can dimensionally oxidize to 5d SYM afterwards (?)

Higher Gauge Theory: Lightening review

To formulate M5-brane theory: Need category theory

Some quotes:

- "We will need to use some very simple notions of category theory, an esoteric subject noted for its difficulty and irrelevance."

  G. Moore and N. Seiberg, 1989

- "We'll only use as much category theory as is necessary. Famous last words..."

  Roman Abramovich

- "Category theory is the subject where you can leave the definitions as exercises."

  John Baez

Fortunately: Categorified Lie algebras very accessible.

NQ-Manifolds

N-manifolds, NQ-manifold

- $\mathbb{N}_0$-graded manifold with coordinates of degree $0, 1, 2, \ldots$

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots$$

manifold   linear spaces

- NQ-manifold: vector field Q of degree 1, $Q^2 = 0$
- Physicists: think ghost numbers, BRST charge, SFT
- Functions on (M, Q) form differential graded algebra
  "Chevalley–Eilenberg algebra"

Examples:

- Tangent algebroid $T[1]M$, $\mathcal{C}^\infty(T[1]M) \cong \Omega^\bullet(M)$, $Q = \mathrm{d}$
- Lie algebra $\mathfrak{g}[1]$, coordinates ξ^a of degree 1:

$$Q = -\tfrac{1}{2} f^\alpha_{\beta\gamma} \xi^\beta \xi^\gamma \frac{\partial}{\partial \xi^\alpha} \quad , \quad \text{Jacobi identity} \Leftrightarrow \; Q^2 = 0$$

Gauge Theory from dga-Morphisms

- Idea: Cartan, More: Strobl et al., Sati, Schreiber, Stasheff
- Local gauge theory: differential forms and Lie algebras
- Unify both in differential graded algebras from Weil algebra:

$$\mathrm{W}(\mathfrak{g}) := \mathcal{C}^\infty(T[1]\mathfrak{g}[1]) = \mathcal{C}^\infty(\mathfrak{g}[1]\oplus\mathfrak{g}[2])~,\quad \sigma:\mathfrak{g}^*[1]\xrightarrow{\cong}\mathfrak{g}^*[2]$$

$$Q|_{\mathcal{C}^\infty(\mathfrak{g}[1])} = Q_{\rm CE}+\sigma~,\quad Q_{\rm CE}\sigma = -\sigma Q_{\rm CE}$$

- Potentials/curvatures/Bianchi identities from dga-morphisms

$$\begin{array}{rcl}
(A,F):\mathrm{W}(\mathfrak{g}) & \longrightarrow & W(M)=\Omega^\bullet(M)\\
\xi^\alpha & \longmapsto & A^\alpha\\
(\sigma\xi^\alpha)=Q\xi^\alpha+\tfrac12 f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma & \longmapsto & F^\alpha=(\mathrm{d}A+\tfrac12[A,A])^\alpha\\
Q(\sigma\xi^\alpha)=-f^\alpha_{\beta\gamma}(\sigma\xi^\alpha)\xi^\beta & \longmapsto & (\nabla F)^\alpha=0
\end{array}$$

- Gauge transformations: homotopies between dga-morphisms
- Topological invariants: invariant polynomials on $\mathfrak{g}$ in $\mathrm{W}(\mathfrak{g})$

$\Rightarrow$ General notion of gauge theory from pairs of dgas

Back to

NQ-manifolds

- $\mathbb{N}_0$-graded manifold with coordinates of degree $0, 1, 2, \ldots$

$$\underbrace{M_0}_{\text{manifold}} \leftarrow \underbrace{M_1 \leftarrow M_2}_{\text{linear spaces}} \leftarrow \ldots$$

- NQ-manifold: vector field Q of degree 1, $Q^2 = 0$
- Functions on (M, Q) form differential graded algebra

- Lie n-algebra or n-term L_∞-algebra:

$$* \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

We shall be interested in Lie 2- and Lie 3-algebras.

Example: Lie 2-Algebras

- Graded vector space: $* \leftarrow W[1] \leftarrow V[2] \leftarrow * \leftarrow \dots$
- Coordinates:
  - w^a of degree 1 on $W[1]$
  - v^i of degree 2 on $V[2]$
- Most general vector field Q of degree 1:

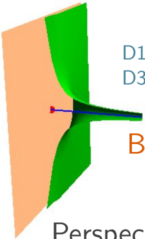
$$Q = -m_i^a v^i \frac{\partial}{\partial w^a} - \tfrac{1}{2} m_{ab}^c w^a w^b \frac{\partial}{\partial w^c} - m_{ai}^j w^a v^i \frac{\partial}{\partial v^j} - \tfrac{1}{3!} m_{abc}^i w^a w^b w^c \frac{\partial}{\partial v^i}$$

- Induces “brackets”/“higher products”:
  - $\mu_1(\tau_i) = m_i^a \tau_a$
  - $\mu_2(\tau_a, \tau_b) = m_{ab}^c \tau_c \ , \quad \mu_2(\tau_a, \tau_i) = m_{ai}^j \tau_j$
  - $\mu_3(\tau_a, \tau_b, \tau_c) = m_{abc}^i \tau_i$
- $Q^2 = 0 \Leftrightarrow$ Homotopy Jacobi identities, e.g.
  - $\mu_1(\mu_1(-)) = 0$: μ_1 is a differential
  - $\mu_2(x, \mu_2(y, z)) + \text{cycl.} = \mu_1(\mu_3(x, y, z))$: Jacobiator
- Analogously: Lie 3-algebras

Which higher Lie algebra to take?

Guidance from BPS self-dual strings

The non-abelian self-dual string



	0	1	2	3	4	5	6
D1	×						×
D3	×	×	×	×			

BPS configuration

Perspective of D3:

Bogomolny monopole eqn.

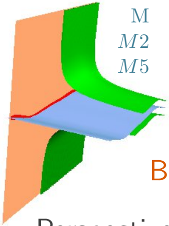
$$F = \nabla^2 = *\nabla\Phi \text{ on } \mathbb{R}^3$$

$\updownarrow$ Nahm transform $\updownarrow$

Perspective of D1:

Nahm eqn.

$$\frac{\mathrm{d}}{\mathrm{d}x^6}X^i + \varepsilon^{ijk}[X^j, X^k] = 0$$



M	0	1	2	3	4	5	6
$M2$	×					×	×
$M5$	×	×	×	×	×	×	

BPS configuration

Perspective of M5:

Abelian Self-dual string eqn.

$$H := \mathrm{d}B = *\mathrm{d}\Phi \text{ on } \mathbb{R}^4$$

$\updownarrow$ genlzd. Nahm transform (?) $\updownarrow$

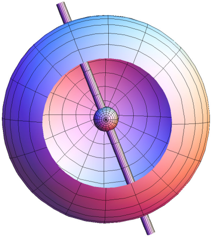
Perspective of M2:

Hoppe-Basu-Harvey eqn. (??)

$$\frac{\mathrm{d}}{\mathrm{d}x^6}X^\mu + \varepsilon^{\mu\nu\rho\sigma}[X^\nu, X^\rho, X^\sigma] = 0$$

Monopoles

- Solution to Bogomolny equation $F = *\nabla\phi$
- Abelian: singular on $\mathbb{R}^3$, Dirac strings
- Principal bundle over S^2
- Non-Abelian: non-singular on $\mathbb{R}^3$

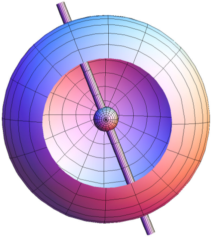


$$\begin{tikzcd}
\mathsf{U}(1) \arrow[r, hook] \arrow[rr, bend left, dashed, "\rho"] & \mathsf{SU}(2) \cong S^3 \arrow[d, "\pi"] \arrow[rr, bend left, dashed, "\pi\times\mathrm{id}"] & \mathsf{SU}(2) \arrow[r, hook] & S^2 \times \mathsf{SU}(2) \arrow[d, "\mathrm{pr}"] \\
 & S^2 \arrow[rr, bend left, dashed, "\mathrm{id}"] & & S^2
\end{tikzcd}$$

abelian, Dirac — non-Abelian, ’t Hooft-Polyakov

$\Rightarrow$ Choose $\mathsf{SU}(2)$, as trivialization possible.

Identifying gauge structure: Self-Dual Strings



Self-Dual Strings

- Abelian: singular on $\mathbb{R}^4$, Dirac strings
- Solution to $H = *\nabla\phi$
- Gerbe over S^3
- Non-Abelian: ?

$$
\begin{tikzcd}
\mathsf{BU}(1) \arrow[r, hook] \arrow[rrr, dashed, bend left, "\rho"] & \mathscr{G}_{\mathrm{F}} \arrow[d, "\pi"] \arrow[rrr, dashed, bend left, "\pi\times\mathrm{id}"] & & \mathscr{G}_{\mathrm{F}} \arrow[r, hook] & (S^3 \rightrightarrows S^3) \times \mathscr{G}_{\mathrm{F}} \arrow[d, "\mathrm{pr}"] \\
& (S^3 \rightrightarrows S^3) \arrow[rrr, dashed, bend left, "\mathrm{id}"] & & & (S^3 \rightrightarrows S^3)
\end{tikzcd}
$$

abelian   non-Abelian ?

$\Rightarrow$ Choose $\mathscr{G}_F$, with 2-group structure: String 2-group
(many other reasons for this)

- String 2-group $\mathscr{G}_F$ and M-theory: long story...
- $\mathscr{G}_F$ is analogue of $\mathsf{Spin}(3) \cong \mathsf{SU}(2)$ from many perspectives
- Lie differentiate (e.g. Demessie, CS (2016))
- Result:
  String Lie 2-algebra $\mathfrak{string}(3) = \left(\mathfrak{su}(2) \xleftarrow{\mu_1=0} \mathbb{R}[1]\right)$ with
  $$Q\xi^\alpha = -\tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma~, \quad Qb = -\tfrac{1}{3!} f_{\alpha\beta\gamma}\xi^\alpha\xi^\beta\xi^\gamma$$
  or
  $$\mu_2(x_1,x_2) = [x_1,x_2]~, \quad \mu_3(x_1,x_2,x_3) = (x_1,[x_2,x_3])$$
  where $x_{1,2,3} \in \mathfrak{su}(2)$.

Remarks:

- Can be defined for any ADE Lie algebra $\mathfrak{g} \rightarrow \mathfrak{string}(\mathfrak{g})$
- Can twist the Weil algebra to $\tilde{W}(\mathfrak{string}(\mathfrak{g}))$ by inv. polynomial

- Recall: Chevalley-Eilenberg algebra of String Lie 2-algebra $\mathfrak{g}$:

$$\mathrm{CE}(\mathfrak{g}) = \mathcal{C}^\infty(\mathbb{R}[2] \to \mathfrak{su}(2)[1]) \ ,$$

$$Q\xi^\alpha = -\tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma \quad \text{and} \quad Qb = \tfrac{1}{3!} f_{\alpha\beta\gamma}\xi^\alpha\xi^\beta\xi^\gamma \ .$$

- Double to Weil algebra:

$$\mathrm{W}(\mathfrak{g}) := \mathcal{C}^\infty(T[1]\mathfrak{g}[1]) = \mathcal{C}^\infty(\mathfrak{g}[1] \oplus \mathfrak{g}[2]) \ , \quad \sigma : \mathfrak{g}^*[1] \xrightarrow{\cong} \mathfrak{g}^*[2]$$

$$Q|_{\mathcal{C}^\infty(\mathfrak{g}[1])} = Q_{\mathrm{CE}} + \sigma \ , \quad Q_{\mathrm{CE}}\sigma = -\sigma Q_{\mathrm{CE}}$$

- Potentials/curvatures/Bianchi identities from dga-morphisms

$$(A, B, F, H) : \mathrm{W}(\mathfrak{g}) \longrightarrow \Omega^\bullet(M) = W(M)$$

$$\xi^\alpha \longmapsto A^\alpha \in \Omega^1(M) \qquad \text{and} \qquad b \longmapsto B \in \Omega^2(M)$$

$$(\sigma\xi^\alpha) = Q\xi^\alpha + \tfrac{1}{2} f^\alpha_{\beta\gamma}\xi^\beta\xi^\gamma \longmapsto F^\alpha = (\mathrm{d}A + \tfrac{1}{2}[A, A])^\alpha$$

$$(\sigma b) = Qb - \tfrac{1}{3!} f_{\alpha\beta\gamma}\xi^\alpha\xi^\beta\xi^\gamma \longmapsto H = \mathrm{d}B - \tfrac{1}{3!}(A, [A, A])$$

- Bianchi identities: $\nabla F = 0$ and $\mathrm{d}H = -\frac{1}{2}(\mathrm{d}A, [A, A])$
- Gauge trafos and Top. invariants derived as above

Field content with values in $\mathfrak{string}(3)$:

$$A \in \Omega^1(\mathbb{R}^4) \otimes \mathfrak{su}(2)\ , \quad B \in \Omega^2(\mathbb{R}^4) \otimes \mathbb{R}\ , \quad \phi \in \Omega^0(\mathbb{R}^4) \otimes \mathbb{R}$$

Need twisted string structures:

$$H = \mathrm{d}B + \tfrac{1}{2}(A, \mathrm{d}A) + \tfrac{1}{3!}(A, [A, A]) = *\mathrm{d}\phi$$

$$\Rightarrow\ dH = (F, F) = *\Box\phi$$

$$F = \mathrm{d}A + \tfrac{1}{2}[A, A] = *F$$

Passes many consistency checks, e.g.

- Nice reduction to monopoles on $\mathbb{R}^3$
- BPS equations for (1,0)-model (more later)

Elementary Solution:

$$A_\mu(x) = \frac{1}{\mathrm{i}}\,\frac{\eta^i_{\mu\nu}\,\sigma_i\,(x - x_0)^\nu}{\rho^2 + (x - x_0)^2}\ , \quad B = 0\ , \quad \varphi = \frac{((x - x_0)^2 - 2\rho^2}{\big((x - x_0)^2 + \rho^2\big)^2}$$

cf. also Akyol, Papadopoulos 2012

The 6d superconformal field theory

Look for candidate theory in the literature and find:

6d (1,0)-model derived from tensor hierarchies
Samtleben, Sezgin, Wimmer (2011)

Open problems with this model:

- Issue 1: Choice of gauge structure unclear
- Issue 2: cubic interactions
- Issue 3: scalar fields with wrong sign kinetic term
- Issue 4: Self-duality of 3-form imposed by hand
- Issue 5: Unclear, how to fulfill "wishlist"

Previous observation:

- Gauge structure is Lie 3-algebra with "extra structure."
  Palmer, CS (2013), Samtleben et al. (2014)

New: Schmidt, CS (2017)

- Idea: use $\mathfrak{string}(\mathfrak{g})$ as gauge structure in this model
- Issue: need suitable notion of inner product for action
- Inner product/cyclic L_∞-algebras $\Leftrightarrow$ symplectic NQ-manifold
- Consequence: Extend twisted $\mathfrak{string}(\mathfrak{g})$ from

$$(\mathfrak{g} \leftarrow \mathbb{R} \leftarrow \mathbb{R}) \;\cong \mathfrak{spin}(\mathfrak{g})$$

to symplectic graded vector space $T^*[2]\mathfrak{string}(\mathfrak{g})$:

$$\begin{array}{ccccc}
\mathbb{R}^* & \xleftarrow{\mu_1=\mathrm{id}} & \mathbb{R}^*[1] & & \mathfrak{g}^*[2] \\
\oplus & & \oplus & & \oplus \\
\mathfrak{g} & & \mathbb{R}[1] & \xleftarrow{\mu_1=\mathrm{id}} & \mathbb{R}[2]
\end{array}$$

- This carries natural inner product
- Can be extended to Lie 3-algebra
- Has necessary extra structure

Field content:

- (1,0) tensor multiplet (ϕ, χ^i, B), values in $\mathbb{R}^2$, $\phi = \phi_s + \phi_r$, ...
- (1,0) vector multiplet (A, λ^i, Y^{ij}), values in $\mathfrak{g} \oplus \mathbb{R}$
- C-field, values in $\mathbb{R} \oplus \mathfrak{g}^*$

Action (schematically):

$$S = \int_{\mathbb{R}^{1,5}} \Big(\mathcal{H}_r \wedge *\mathcal{H}_s + \mathrm{d}\phi_r \wedge *\mathrm{d}\phi_s - *\bar{\chi}_r \not{\partial} \chi_s + \mathcal{H}_s \wedge *(\bar{\lambda}, \gamma_{(3)} \lambda) + *(Y, \bar{\lambda})\chi_s$$
$$+ \phi_s \big((\mathcal{F}, *\mathcal{F}) - *(Y, Y) + *(\bar{\lambda}, \not{\nabla} \lambda) \big) + (\bar{\lambda}, \mathcal{F}) \wedge *\gamma_{(2)} \chi_s$$
$$+ \mu_1(C) \wedge \mathcal{H}_s + B_s \wedge (\mathcal{F}, \mathcal{F}) + B_s \wedge ([A, A], [A, A]) \Big)$$

This solves problems 1 and 2:

- Choice of gauge structure for ADE-(2,0)-theories clear.
- No cubic interaction term for scalar fields

Adding Pasti-Sorokin-Tonin-type action:

- Recall: PST action has self-duality of H as equation of motion
- Bosonic part of (1,0)-theory was PST completed
  Bandos, Sorokin, Samtleben (2013)
- Full PST action announced, never appeared (not possible?)
- With string structure, construction possible and simplifies

Adding matter fields:

- Add hypermultiplet to get fields of (2,0)-tensor multiplet
- General construction and couplings discussed
  Samtleben, Sezgin, Wimmer (2012)
- Can make concrete choices with twisted string structures

$\Rightarrow$ A (1,0)-theory in 6d satisfying many of the "wishlist" items.

Consistency checks

- ✓ Contain an interacting, self-dual 2-form gauge potential
- ✓ Based on a sound mathematical foundation: higher bundles
- ✓ Field content of the $(2,0)$-theory, $\mathcal{N}=(1,0)$ supersymmetric
- ✓ Gauge structure natural, match some expectations (ADE, ...)
- ✓ Non-trivial coupling, interacting field theory
- ✓ Possible restriction to free $\mathcal{N}=(2,0)$ tensor multiplet
- ✓ contains the non-abelian self-dual string soliton as BPS state
- $\rightarrow$ Reduction to 4d SYM theory with ADE gauge algebras
- $\rightarrow$ and to 3d Chern–Simons-matter models with discrete coupling
- ? match expected moduli space of $\mathcal{N}=(2,0)$-theory

Consistency check: Reduction to SYM theory

Crucial consistency check: Reduction to D-branes/SYM theory

$$
\begin{aligned}
S = \int_{\mathbb{R}^{1,5}} \Big(& \langle \mathcal{H}, *\mathcal{H} \rangle + \langle \mathrm{d}\phi, *\mathrm{d}\phi \rangle - *\langle \bar{\chi}, \partial\!\!\!/ \chi \rangle + \mathcal{H}_s \wedge *(\bar{\lambda}, \gamma_{(3)}\lambda) + *(Y, \bar{\lambda})\chi_s \\
& + \phi_s \big((\mathcal{F}, *\mathcal{F}) - *(Y, Y) + *(\bar{\lambda}, \nabla\!\!\!\!/ \lambda) \big) + (\bar{\lambda}, \mathcal{F}) \wedge *\gamma_{(2)}\chi_s \\
& + \mu_1(C) \wedge \mathcal{H}_s + B_s \wedge (\mathcal{F}, \mathcal{F}) + B_s \wedge ([A, A], [A, A]) \Big)
\end{aligned}
$$

- Start from ADE-String Lie 3-algebra
- Anticipate 4d gauge couplings:

$$\tau = \tau_1 + \mathrm{i}\tau_2 = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{g_{\mathrm{YM}}^2} \ ,$$

- VEVs from compactification on T^2 along x^9 and x^{10}

$$\langle \phi_s \rangle = -\frac{1}{32\pi^2} \frac{\tau_2}{R_9 R_{10}} \quad \text{and} \quad \langle B_s \rangle = \frac{1}{16\pi^2} \frac{\tau_1}{R_9 R_{10}}$$

- Strong coupling expansion around VEVs (cf. M2 $\rightarrow$ D2)
- $\Rightarrow$ 4d $\mathcal{N} = 4$ SYM with ADE-gauge group and θ-term

Additional consistency check: Reduction to M2-brane models

- Replace $\mathbb{R}^{1,5}$ by $\mathbb{R}^{1,2} \times S^3$.
- Assumptions:
  - String Lie 3-algebra of $\mathfrak{su}(n) \times \mathfrak{su}(n)$
  - A trivial on S^3, non-trivial on $\mathbb{R}^{1,2}$
  - B trivial on $\mathbb{R}^{1,2}$
  - B encodes abelian gerbe with DD class k on S^3.
- Recall: $\mathcal{H} = \mathrm{d}B + \mathrm{cs}(A)$
- Then we get the integer Chern–Simons coupling:

$$\mathcal{H} \wedge *\mathcal{H} \to k \mathrm{vol}_{S^3} \; \mathrm{cs}(A)$$

$$\int_{\mathbb{R}^{1,5}} \mathcal{H} \wedge *\mathcal{H} \to k \int_{\mathbb{R}^{1,2}} \mathrm{cs}(A)$$

- Altogether: Chern–Simons matter theory of ABJM type.
- Note: This theory has $\mathcal{N} = 4$, different potential from ABJM.

Problems and potential solutions

Open Problems

Our model is not the desired (2,0)-theory!

Problems:

- Free Yang–Mills multiplet contradicts $\mathcal{N} = (2,0)$ SUSY
- Moduli space of vacua is not that of multiple M5-branes
- PST mechanism relies on $\phi_s > 0$
- Scalar field with wrong sign kinetic term
- Model not compatible with categorical equivalence

Turn problems into hints of solution:

- Scalar field with wrong sign kinetic term
  (rigid feature of Samtleben et al. model)
- Model not compatible with categorical equivalence
  (rigid feature of Samtleben et al. model)

Last point: the model of Samtleben et al. is too rigid:

$$(X_r)_s{}^t = f_{rs}^t + d_{rs}^t = f_{[rs]}^t + d_{(rs)}^t$$

Next steps/work in progress:

- String 2-algebra $\rightarrow$ Lie 2-algebras with right branching
  L Schmidt & CS, arXiv 1805.?????
- Understand twisted Weil algebras and categorical equivalence
- Rederive SUSY action in bigger picture

Summary:

- Higher gauge theory classically underlies M-theory
- Higher analogue of $\mathsf{SU}(2)$ is $\mathsf{String}(3)$
- There is non-abelian self-dual string
- There is classical action with many of desired features
- However: Clear differences to (2,0)-theory

Soon to come:

▷ Understand generalization of String Structure (WIP)
▷ Understand Categorical Equivalence, Higher Twists (WIP)
▷ Study $\mathcal{N} = (1,0)$-models (next on our list)
▷ Link to categorified integrability, fuzzy S^3, etc. (future)
▷ Better understanding of M-theory (far future)

Announcement

LMS/EPSRC Durham Symposium

Higher Structures in M-Theory

12.-18. August 2018

Topics covered:

- Higher Differential Geometry and Higher Lie Theory
- Higher Gauge Theory
- Higher Structures in M- and F-Theory
- Double and Exceptional Field Theory and Duality Symmetric String and M-Theory
- Higher (Pre)Quantisation

More: Contact me or google “Durham symposium” for webpage.

Towards an M5-brane model: A 6d superconformal field theory

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ENS/UPMC Seminars, Paris, 15.5.2018