(2,0)-theory + Higher Gauge Theory.

Gauge Theory

- E.g. Maxwell: electric/magnetic fields E/B

$\rightarrow$ spacetime $\rightarrow$ $F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & B_y \\ -\vdots & & 0 & B_x \\ & & & 0 \end{pmatrix}$ $\quad F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$

summation.

Maxwell: $dF = 0$, $*d*F = 0$ $\rightarrow$ Poincaré lemma $F = dA$

note: $A \to A + d\alpha$ leaves α invariant.

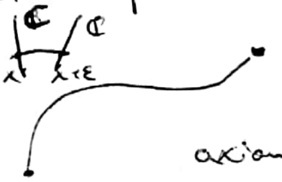
A basic field but redundancy in description.

- Mathematically: fields ψ with values in $\mathbb{C}$ (describing e.g. electrons) $\in U(1)$

$\rightarrow$ need to define parallel transport. $\mathbb{C}$ / $\mathbb{C}$ at x, $x+\varepsilon$

$\partial\psi \sim \lim_{\varepsilon\to 0} \frac{\psi(x+\varepsilon)-\psi(x)}{\varepsilon}$

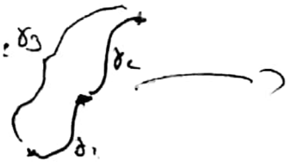
e.g. to define energy of field etc.



Curve $\gamma \to$ group element $g(\gamma)$

ψ transported along γ: $\psi \to g(\gamma)\cdot\psi$

axioms: γ_3, γ_2, γ_1



$g(\gamma_1)\cdot g(\gamma_2) = g(\gamma_3)$

Functor $\Pi M \longrightarrow \begin{matrix} G \\ \Downarrow \\ * \end{matrix}$

infinitesimally: $A \in \Omega^1(M) \otimes \mathrm{Lie}(G)$ $\quad g(\gamma) = \mathcal{P}\exp \int_\gamma A$

- Globally:

P: principal fibre bundle

M: manifold.

G: Lie group acting properly on P

global 1-form $\omega \in \Omega^1(P) \otimes \mathrm{Lie}(G)$

+ conditions: project $T_u P$ onto $V_u P \cong \mathfrak{g}$

$A_i = \sigma_i^* \omega$.

- Relevance:

Physics: · EM · GR · Standard Model · String/D-branes.

Math: · Instantons, Hitchin, ADHM · Moduli spaces · Topological invariants · ⋮

Global/Dual

~~Local description~~

$\mathfrak{g}$ Lie algebra

$\mathcal{N}=4$ SYM + descendants $\longrightarrow$ multiple M5, (2,0)-theory + descendants.

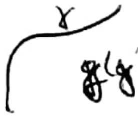
Dbranes + dualities $\uparrow$ M5-branes

Mysterious, very important.

What is known: (2,0)-theory:

- abelian theory: $B \in \Omega^2(M)$ $\quad H = dB = *H$
- SUSY field content
- observables: Wilson surfaces.
- ~~Theory of M5-branes~~

$\rightarrow$ Higher gauge theory.



$g(\gamma) = P\exp\int_\gamma A \Rightarrow g(\sigma) = \exp\int_\sigma B$

Naive problems with consistency $\rightarrow$ category theory to fix.
for non-abelian.

L_∞-algebras.

Generally: $\mathfrak{g} = \mathrm{Lie}(G)$, $\quad A \in \Omega^1(M)\otimes\mathfrak{g}$

higher:

paths + surfaces, need two objects $A \in \Omega^1(M)\otimes\mathfrak{g}_0$ $\quad B \in \Omega^2(M)\otimes\mathfrak{g}_{-1}$

generalise Lie algebra: dg Lie: vector space $\underline{\mathfrak{g}} = \bigoplus_{i\in\mathbb{Z}} \mathfrak{g}_i$

d of degree 1

graded Lie bracket $[-,-]$ of degree 0

L_∞-alg: $\quad L = \bigoplus_{i\in\mathbb{Z}} L_i \qquad \mu_i: L^{\wedge i} \to L$

$$\sum_{\text{cycl}} [[a,b],c] = 0$$

$$\downarrow$$

$$\sum_{\text{perm}} \pm \mu_{i+1}(\mu_j(-,\dots,-),-,\dots,-) = 0$$

$\mu_1^2 = 0 \qquad \mu_1(\mu_2(a,b)) = \mu_2(\mu_1(a),b) + (-1)^{|a|}\mu_2(a,\mu_1(b))$

violate Jacobi $\longrightarrow$ $\mu_2(\mu_2(a,b),c) = \mu_1(\mu_3(a,b,c)) + \mu_3(\mu_1(a),b,c) + \dots$

Notion: equivalence $\simeq$ quasi iso:

Thm (1) Any L_∞ quasi-iso to strict one: $\quad \mu_i = 0$ for $i \geq 3$

(2) — " — minimal one: $\quad \mu_1 = 0$

LMC-Theory:

$a \in L_1$: gauge potential

$f := \sum_{i \geq 1} \frac{1}{i!} \mu_i(a, \dots, a)$ curvature

gauge trafos from <u>flat</u> homotopies.
$\to f$ defines δa

e.g. $L = \Omega^\bullet$, $\mu_1 = d$

$a \in \Omega^1$ $\quad f = da$

more generally: $\hat{L} = \Omega^\bullet_{(M)} \otimes L$ degrees add

$$\hat{\mu}_1 = d \otimes 1 + 1 \otimes \mu_1$$
$$\hat{\mu}_i = \pm \mu_i \qquad i \geq 2$$

yields higher gauge theory with gauge algebra L.

e.g. $L = L_{-1} \oplus L_0$:

$a = A + B \in \hat{L}_1$, $A \in \Omega^1(M) \otimes L_0$
$\qquad B \in \Omega^2(M) \otimes L_{-1}$

$f = \mathcal{F} + H \in \hat{L}_2$

$$\mathcal{F} = dA + \tfrac{1}{2}\mu_2(A, A) + \mu_1(B)$$
$$H = dB + \mu_2(A, B) + \tfrac{1}{3!}\mu_3(A, A, A)$$

gauge trafo
~~$\tilde{A} = A + \dots\, d\alpha$~~

$\alpha + \Lambda \in \hat{L}_0$ $\qquad \alpha \in \Omega^0(M) \otimes L_0$
$\qquad \Lambda \in \Omega^1(M) \otimes L_{-1}$

$$A \xmapsto{\Gamma_{\alpha+\Lambda}} A + d\alpha + \mu_2(A, \alpha) + \mu_1(\Lambda)$$
$$B \mapsto B + d\Lambda + \mu_2(A, \Lambda) + \mu_2(\alpha, \mathcal{F})$$

Issue: gauge trafos only close if $\mathcal{F} = 0$

$$[\Gamma_{\alpha_1+\Lambda_1}, \Gamma_{\alpha_2+\Lambda_2}] = \Gamma_{\alpha_3+\Lambda_3} + \mu_3(\mathcal{F}, A, \alpha)$$

Switch to sheaf one; still, finite trafos only glue together, if $\mathcal{F} = 0$.

Also
- $H = * H$ only covariant if $\mathcal{F} = 0$
- parallel transport only rep-inv. if $\mathcal{F} = 0$.

~~Why is this~~ This <u>is bad</u>!

Note: minimal case: $\mu_1 = 0$

$$\mathcal{F} = dA + \tfrac{1}{2}[A,A] = 0 \quad \to \text{Poincaré: locally gauge } A \text{ to } 0.$$

$$H = dB + \mu_2(A,B) + \mu_3(A,A,A)$$

A goes away, $H = dB$, <u>abelian</u> thing.

Want: non-abelian,

How to fix. Turns out:

- More freedom in definition of H for certain ...

$$\cancel{H \to H - K}$$

e.g. $\mathfrak{string}(3)$:

$$\mathbb{R} \to \mathfrak{su}(2) = \mathfrak{spin}(3)$$

$$L_{-1} \to L_0$$

$$\mu_1 = 0$$

$$\mu_2 = [-,-]$$

$$\mu_3 = (-,[-,-])$$

$$\mathcal{F} = dA + \tfrac{1}{2}[A,A]$$

$$H = dB + \tfrac{1}{3!}\mu_3(A,[A,A])$$

- Freedom!

$$H \to H - \tfrac{1}{2}(A,\mathcal{F}) = dB + \tfrac{1}{2}(A,dA) + \tfrac{1}{3!}(A,[A,A])$$

$\to$ no more fake curvature. $\to$ $\boxed{\text{String Structures}}$

related to Spin structures on loop space

Conclusions

- YM + CS have <u>same</u> kinematical data, gauge connections on principal bdls

- higher CS easy to define.
- higher YM: (2,0) requires modification.