Extended Riemannian Geometry and Double Field Theory

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Based on:

- arXiv:1611.02772 with Andreas Deser

Aim: Clarify mathematical structures behind Double Field Theory

Double Field Theory (DFT)

- target space formulations useful (SUSY YM, SUGRA) ⎫
- T-duality very interesting   ⎬ DFT
                                                     ⎭

  Tseytlin, Siegel, Hull, Zwiebach, . . .

- Success, e.g.: DFT action on $\mathbb{R}^{2D}$:

$$S = \int \mathrm{d}^{2D}x \; \mathrm{e}^{-2d} \left(\tfrac{1}{8}\mathcal{H}_{MN}\partial^M\mathcal{H}_{KL}\partial^N\mathcal{H}^{KL} - \tfrac{1}{2}\mathcal{H}_{MN}\partial^M\mathcal{H}_{KL}\partial^L\mathcal{H}^{KN} \right.$$

$$\left. - 2\partial^M d\partial^N\mathcal{H}_{MN} + 4\mathcal{H}_{MN}\partial^M d\partial^N d \right)$$

  $\mathcal{H}_{MN}$: generalized metric, d: DFT dilaton

- Reproduces SUGRAs previously not derived from string theory
- sensible? strange truncation of string modes? seems to work

Clarifying the mathematical structures underlying Double Field Theory.

Manifest invariance under T-duality, coordinates $x^M = (x^\mu, x_\mu)$:

- g and B transform differently, combine in generalized metric:

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\kappa} g^{\kappa\lambda} B_{\lambda\nu} & B_{\mu\kappa} g^{\kappa\nu} \\ -g^{\mu\kappa} B_{\kappa\nu} & g^{\mu\nu} \end{pmatrix}$$

- Level matching: Fields ϕ in DFT satisfy $\quad \Box\phi = 0$

- Algebra of fields: strong section condition:

$$\partial^M \phi \partial_M \psi = 0$$

- Generalized Lie derivative wrt. $X = X^\mu \frac{\partial}{\partial x^\mu} + X_\mu \frac{\partial}{\partial x_\mu}$:

$$\hat{\mathcal{L}}_X \mathcal{H}_{MN} = X^P \partial_P \mathcal{H}_{MN} + (\partial_M X^P - \partial^P X_M) \mathcal{H}_{PN} + (\partial_N X^P - \partial^P X_N) \mathcal{H}_{MP}$$

- $\hat{\mathcal{L}}_X$ form Lie algebra, but not representation of Lie algebra

$$\hat{\mathcal{L}}_X \hat{\mathcal{L}}_Y - \hat{\mathcal{L}}_Y \hat{\mathcal{L}}_X = \mathcal{L}_{\mu_2(X,Y)} \ ,$$

- $\mu_2(X, Y)$ known as C-bracket, actually: part of L_∞-algebra

Issues to address in a mathematical formulation:

- Issue 1: Section condition is ugly/coordinate dependent:
$$\partial^M \phi \; \partial_M \psi = 0$$

- Issue 2: Section condition is often too strong

- Issue 3: Reasonable version of Doubled Riemannian Geometry
$$\widehat{\Gamma}_{MNK} = -2(P\partial_M P)_{[NK]} - 2(\bar{P}_{[N}{}^P \bar{P}_{K]}{}^Q - P_{[N}{}^P P_{K]}{}^Q)\partial_P P_{QM}$$
$$+ \frac{4}{D-1}(P_{M[N}P_{K]}{}^Q + \bar{P}_{M[N}\bar{P}_{K]}{}^Q)(\partial_Q d + (P\partial^P P)_{[PQ]})$$

- Issue 4: Global formulation of DFT

- Issue 5: Same for Exceptional field theory (M-theory analogue)

- Massless string modes: metric g, 2-form B, dilaton ϕ.
- Recall: B-field belongs to gerbe/categorified principal bundle
- Generalized Geometry: Courant/Lie 2-algebroid $TM \oplus T^*M$
- Captures Lie 2-algebra of symmetries (gauge+diffeos)
- In particular: Courant/Dorfman brackets
- Similarly: GR coupled to n-form fields
- Picture allows for an extension to pre-Lie 2-algebroids
- Lie 2-algebra with C- and D-brackets
- Categorified Jacobi relations $\Leftrightarrow$ weakened section condition
- Fully algebraic and coordinate independent picture
- Know how to twist pre-Lie 2-algebroids
- Potential global picture (work in progress)

Part I : Generalized Geometry

Generalized Geometry provides a nice geometric description of the B-field.

Ingredients: massless string modes: metric g, 2-form B, dilaton ϕ.

Gerbes

- Well-known: B belongs to connective structure of a gerbe

  Gawedzki 1987, Freed&Witten 1999

- Manifold M, cover $Y = \sqcup_i U_i \twoheadrightarrow M$, $Y^{[2]} = \sqcup_{i,j} U_i \cap U_j$, ...

- Principal $\mathsf{U}(1)$-bundle over M:

$$\mathsf{U}(1) \to P \to M \ , \quad g_{ij} g_{jk} = g_{ik} \ , \quad A_i = g_{ij}^{-1}(A_j + \mathrm{d})g_{ij}$$

- $\mathsf{U}(1)$-gerbe $\mathscr{G}$ over M:

$$
\begin{array}{ccccccc}
\mathsf{U}(1) & \longrightarrow & \mathscr{G}_1 & \longrightarrow & Y^{[2]} & \overset{\cong}{\longrightarrow} & M \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
* & & \mathscr{G}_0 & & Y & & M
\end{array}
$$

$$h_{ijk} h_{ik\ell} = h_{ij\ell} h_{jk\ell} \ , \ A_{ij} - A_{ik} + A_{jk} = \mathrm{d}\log(h_{ijk}) \ , \ B_i - B_j = \mathrm{d}A_{ij}$$

- Categorified spaces $\Rightarrow$ categorified symmetries!

Generalized Geometry provides a nice geometric description of the B-field.

Ordinary geometry: "Local" Atiyah Algebroid for U(1)-bundle

$$0 \to M \times i\mathbb{R} \xrightarrow{\hookrightarrow} TM \oplus i\mathbb{R} \xrightarrow{\mathrm{pr}} TM \to 0$$
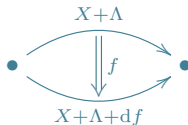
- Gauge potential $A : TM \to i\mathbb{R}$ is defined by section of pr
- Infinitesimal symm. (diffeos+gauge): sections of $TM \oplus i\mathbb{R}$

Generalized Geometry: Courant algebroid for U(1)-gerbe

$$0 \to T^*M \xrightarrow{\hookrightarrow} TM \oplus T^*M \xrightarrow{\mathrm{pr}} TM \to 0$$

- $g + B : TM \to T^*M$ again from section of pr
- Infinitesimal symm. (diffeos+gauge): sections of $TM \oplus T^*M$

not complete picture:

NQ-manifolds, known from BRST quantization, provide very useful language.

N-manifolds, NQ-manifold

- $\mathbb{N}$-graded manifold with coordinates of degree $0, 1, 2, \ldots$

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots$$

manifold   linear spaces

- NQ-manifold: vector field Q of degree 1, $Q^2 = 0$
- Physicists: think ghost numbers, BRST charge, SFT
- symplectic NQ-manifold: ω nondegenerate, closed, $\mathcal{L}_Q \omega = 0$

Examples:

- Tangent algebroid $T[1]M$, $\mathcal{C}^\infty(T[1]M) \cong \Omega^\bullet(M)$, $Q = \mathrm{d}$
- Lie algebra $\mathfrak{g}[1]$, coordinates ξ^a of degree 1:

$$Q = -\tfrac{1}{2} f^c_{ab} \xi^a \xi^b \frac{\partial}{\partial \xi^c} \qquad \text{Jacobi identity} \Leftrightarrow Q^2 = 0$$

L_∞-Algebras, Lie 2-Algebras

NQ-manifolds provide an easy definition of L_∞-algebras.

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Lie n-algebroid or n-term L_∞-algebroid:

$$M_0 \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Lie n-algebra or n-term L_∞-algebra:

$$* \leftarrow M_1 \leftarrow M_2 \leftarrow \ldots \leftarrow M_n \leftarrow * \leftarrow * \leftarrow \ldots$$

Important example of NQ-manifold: Lie 2-algebra

$$* \leftarrow W[1] \leftarrow V[2] \leftarrow * \leftarrow \ldots \ , \quad \text{coords. } w^a, \ v^i$$

$$Q = -m_i^a v^i \frac{\partial}{\partial w^a} - \frac{1}{2} m_{ab}^c w^a w^b \frac{\partial}{\partial w^c} - m_{ai}^j w^a v^i \frac{\partial}{\partial v^j} - \frac{1}{3!} m_{abc}^i w^a w^b w^c \frac{\partial}{\partial v^i} \ ,$$

$$\mu_1(\tau_i) = m_i^a \tau_a \ , \quad \mu_2(\tau_a, \tau_b) = m_{ab}^c \tau_c \ , \quad \ldots \ , \quad \mu_3(\tau_a, \tau_b, \tau_c) = m_{abc}^i \tau_i$$

Examples of symplectic NQ-manifolds:

- Symplectic manifolds:

$$(M, \omega) \ , \quad Q = 0$$

- Poisson manifolds:

$$T^*[1]M \ , \quad \omega = \mathrm{d}p_\mu \wedge \mathrm{d}x^\mu \ , \quad Q = \pi^{\mu\nu} p_\mu \frac{\partial}{\partial x^\nu}$$

The L_∞-algebra of a symplectic L_∞-algebroid

Every symplectic L_∞-algebroid comes with an L_∞-algebra.

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An underappreciated/widely unknown fact:

All symplectic Lie n-algebroids $\mathcal{M}$ come with Lie n-algebra $\mathsf{L}(\mathcal{M})$:

$$\mathcal{C}_0^\infty(\mathcal{M}) \xrightarrow{Q} \quad \mathcal{C}_1^\infty(\mathcal{M}) \xrightarrow{Q} \quad \ldots \xrightarrow{Q} \quad \mathcal{C}_{n-2}^\infty(\mathcal{M}) \xrightarrow{Q} \quad \mathcal{C}_{n-1}^\infty(\mathcal{M})$$

$$\mathsf{L}_{n-1}(\mathcal{M}) \xrightarrow{Q} \quad \mathsf{L}_{n-2}(\mathcal{M}) \xrightarrow{Q} \quad \ldots \xrightarrow{Q} \quad \mathsf{L}_1(\mathcal{M}) \xrightarrow{Q} \quad \mathsf{L}_0(\mathcal{M})$$

$$\mu_1(\ell) = \begin{cases} 0 & \ell \in \mathcal{C}_{n-1}^\infty(\mathcal{M}) = \mathsf{L}_0(\mathcal{M}) \\ Q\ell & \text{else} \end{cases}$$

$$\mu_2(\ell_1, \ell_2) = \tfrac{1}{2}\left(\{\delta\ell_1, \ell_2\} \pm \{\delta\ell_2, \ell_1\}\right), \quad \delta(\ell) = \begin{cases} Q\ell & \ell \in \mathsf{L}_0(\mathcal{M}) \\ 0 & \text{else} \end{cases}$$

$$\mu_3(\ell_1, \ell_2, \ell_3) = -\tfrac{1}{12}\left(\{\{\delta\ell_1, \ell_2\}, \ell_3\} \pm \ldots\right)$$

Roytenberg, Rogers, Fiorenza, Getzler

"Higher derived brackets" (Kosmann-Schwarzbach, Voronov, ...)

Important class of examples next.

The key geometric structure behind generalized geometry is locally $T^*[2]T[1]M$.

The symplectic NQ-manifold $T^*[2]T[1]M$

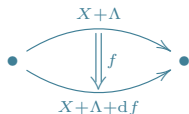Local description, choose $\mathbb{R}^D$ as base manifold.

$$T^*[2]T[1]\mathbb{R}^D = \mathbb{R}^D \ \oplus \ \mathbb{R}^D[1] \ \oplus \ \mathbb{R}^D[1] \ \oplus \ \mathbb{R}^D[2]$$
$$\phantom{T^*[2]T[1]\mathbb{R}^D =} x^\mu \qquad \xi^\mu \qquad\quad \zeta_\mu \qquad\quad p_\mu$$

$$Q = \xi^\mu \frac{\partial}{\partial x^\mu} + p_\mu \frac{\partial}{\partial \zeta_\mu} \qquad \omega = \mathrm{d}x^\mu \wedge \mathrm{d}p_\mu + \mathrm{d}\xi^\mu \wedge \mathrm{d}\zeta_\mu$$

Lie 2-algebra of $\mathcal{M} := T^*[2]T[1]\mathbb{R}^D$: symmetry algebra of gerbe!

- $\mathcal{C}_0^\infty(\mathcal{M}) \to \mathcal{C}_1^\infty(\mathcal{M}) = \mathsf{L}_1(\mathcal{M}) \to \mathsf{L}_0(\mathcal{M})$
- $\mathcal{C}_1^\infty(\mathcal{M}) = \mathsf{L}_0(\mathcal{M}) \cong \Gamma(TM \oplus T^*M)$
- Dorfman: $\nu_2(\ell_1, \ell_2) = \{\delta\ell_1, \ell_2\}$
- Courant: $\mu_2(\ell_1, \ell_2) = \frac{1}{2}\big(\{\delta\ell_1, \ell_2\} \pm \{\delta\ell_2, \ell_1\}\big)$

Part of Larger Family: GR + n-form potentials

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The above picture is part of a larger story, involving GR coupled to n-form potentials.

More generally: Couple GR to n-form gauge potential

Vinogradov algebroids $\mathcal{V}_n(M)$

- Locally as a vector bundle: $\mathcal{V}_n(M) := T^*[n]T[1]M$
- coords: $(x^\mu, \xi^\mu, \zeta_\mu, p_\mu)$ of degrees $(0, 1, n-1, n)$
- Homological vector field: $Q = \xi^\mu \frac{\partial}{\partial x^\mu} + p_\mu \frac{\partial}{\partial \zeta_\mu}$
- Symplectic form: $\omega = \mathrm{d}x^\mu \wedge \mathrm{d}p_\mu + \mathrm{d}\xi^\mu \wedge \mathrm{d}\zeta_\mu$

- Geometric picture: principal n-bundle or $n-1$-gerbe
- diffeos+gauge: degree $n-1$ functions of $\mathcal{V}_n(M)$.
- Higher Courant/Dorfman brackets
- Full Symmetrie Lie n-algebra: Lie n-algebra of $\mathcal{V}_n(M)$

How to extend this to Double Field Theory?

Part II : Double Field Theory / Extended Geometry

There are two properties that offer themselves to a weakening.

> **Requirements**
>
> - Geometry built from spacetime
> - Symmetry Lie n-algebra structure from derived brackets
> - Reduction to Vinogradov algebroids of Generalized Geometry
> - a few further points, related to global picture

Need to keep: graded vector bundle, symplectic form, $|Q| = 1$

Note: $Q^2 = 0$ not necessary everywhere for L_∞-algebra:

$$\mathsf{L}_{n-1}(\mathcal{M}) \xrightarrow{Q} \mathsf{L}_{n-2}(\mathcal{M}) \xrightarrow{Q} \ldots \xrightarrow{Q} \mathsf{L}_1(\mathcal{M}) \xrightarrow{Q} \mathsf{L}_0(\mathcal{M}) \longrightarrow 0$$

Instead: something like $\{Q^2 -, -\} = 0$.

(Exceptional field theory: Q does not have to be vector field.)

All relevant notions can be reasonably extended.

Definition: Symplectic pre-NQ-Manifold of degree n

Symplectic N-manifold $(\mathcal{M}, \omega)$ of degree n, i.e. $|\omega| = n$, compatible vector field Q of degree 1, i.e. $|Q| = 1$ and $\mathcal{L}_Q \omega = 0$.

Definition: L_∞-structure

A subset $\mathsf{L}(\mathcal{M})$ of the functions $\mathcal{C}^\infty(\mathcal{M})$ such that the derived brackets close and form an L_∞-algebra.

Theorem

$(\mathcal{M}, \omega)$ of degree 2.
$\mathsf{L}(\mathcal{M}) = \mathsf{L}_1(\mathcal{M}) \oplus \mathsf{L}_0(\mathcal{M}) \subset \mathcal{C}^\infty(\mathcal{M})$, derived brackets close.
$\mathsf{L}(\mathcal{M})$ is L_∞-structure iff for all $f, g \in \mathsf{L}_1(\mathcal{M})$, $X, Y, Z \in \mathsf{L}_0(\mathcal{M})$:
$$\{Q^2 f, g\} + \{Q^2 g, f\} = 0 , \quad \{Q^2 X, f\} + \{Q^2 f, X\} = 0$$
$$\{\{Q^2 X, Y\}, Z\}_{[X,Y,Z]} = 0$$

This framework is readily applied to reproduce DFT's strong section condition.

- Start: $M = \mathbb{R}^D$, double: T^*M, coords. $x^M = (x^\mu, x_\mu)$
- $\mathcal{V}_2(T^*M) = T^*[2]T[1](T^*M)$, coords. $(x^M, \xi^M, \zeta_M, p_M)$
- $\omega = \mathrm{d}x^M \wedge \mathrm{d}p_M + \mathrm{d}\xi^M \wedge \mathrm{d}\zeta_M$, $\quad Q = \sqrt{2}\big(\xi^M \frac{\partial}{\partial x^M} + p_M \frac{\partial}{\partial \zeta_M}\big)$
- Crucial to GenGeo/DGT: T-duality group $\mathsf{O}(D,D)$
- Reduce structure group $\mathsf{GL}(2D, \mathbb{R})$ to $\mathsf{O}(D,D)$ by introducing

$$\eta_{MN} = \eta^{MN} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

- New coordinates:

$$\theta^M = \frac{1}{\sqrt{2}}(\xi^M + \eta^{MN}\zeta_N) \qquad \beta^M = \frac{1}{\sqrt{2}}(\xi^M - \eta^{MN}\zeta_N)$$

- Polarize by putting $\beta^M \overset{!}{=} 0$:

$$Q = \theta^M \frac{\partial}{\partial x^M} + p_M \eta^{MN} \frac{\partial}{\partial \theta^N}, \quad Q^2 = p_M \eta^{MN} \frac{\partial}{\partial x^N} \neq 0$$

- We obtain a symplectic pre NQ-manifold, $\mathcal{E}_2(M)$.

This framework is readily applied to reproduce DFT's strong section condition.

$$\mathcal{E}_2(\mathbb{R}^D) \quad = \quad \mathbb{R}^{2D} \quad \oplus \quad \mathbb{R}^{2D}[1] \quad \oplus \quad \mathbb{R}^{2D}[2]$$

$$x^M \qquad\qquad \theta^M \qquad\qquad p_M$$

$$Q = \theta^M \frac{\partial}{\partial x^M} + p_M \eta^{MN} \frac{\partial}{\partial \theta^N} \qquad \omega = \mathrm{d}x^M \wedge \mathrm{d}p_M + \tfrac{1}{2}\eta_{MN}\mathrm{d}\theta^M \wedge \mathrm{d}\theta^N$$

Proposition

Let $\mathsf{L} = \mathsf{L}_0 \oplus \mathsf{L}_1$ be L_∞-structure on $\mathcal{E}_2(\mathbb{R}^D)$, $f, g \in \mathsf{L}_1$ and $X = X_M \theta^M, Y = Y_M \theta^M, Z = Z_M \theta^M \in \mathsf{L}_0$. Then:

$$\{Q^2 f, g\} + \{Q^2 g, f\} = 2\partial^M f \; \partial_M g = 0$$

$$\{Q^2 X, f\} + \{Q^2 f, X\} = 2\partial^M X \; \partial_M f = 0$$

$$\{\{Q^2 X, Y\}, Z\}_{[X,Y,Z]} = 2\theta^L \big((\partial^M X_L)(\partial_M Y^K) Z_K\big)_{[X,Y,Z]} = 0$$

Note: Fulfilled for $\partial^M \phi \; \partial_M \psi = 0 \Rightarrow$ Weakened section condition

The picture we have:

- Symmetries of DFT are described by L_∞-structure on $\mathcal{E}_2(\mathbb{R}^D)$
- Extended Geometry $\mathcal{E}_2(\mathbb{R}^D)$: polarization of GenGeo
- Reduce Ext. Geo. $\to$ GenGeo: Choice of L_∞-structure
- Example:

$$\mathrm{L} = \left\{ F \in \mathcal{C}^\infty(\mathcal{E}_2(\mathbb{R}^D)) \,|\, \frac{\partial}{\partial x_\mu} F = 0 \right\}$$

$$Q = \theta^\mu \frac{\partial}{\partial x_\mu} + p_\mu \frac{\partial}{\partial \theta^\mu} \qquad \omega = \mathrm{d}x^\mu \wedge \mathrm{d}p_\mu + \mathrm{d}\theta^\mu \wedge \mathrm{d}\theta_\mu$$

$\Rightarrow$ Vinogradov algebroid $\mathcal{V}_2(\mathbb{R}^D)$ of Generalized Geometry

Action of Symmetries

An action of the symmetry Lie n-algebra can be defined.

Observations:

- Action of Lie algebra $\mathfrak{g}$ on manifold M: hom. $\mathfrak{g} \to \mathfrak{X}(M)$
- N-manifold $\mathcal{M}$, $\mathfrak{X}(\mathcal{M})$ is $\mathbb{N}$-graded Lie algebra, is L_∞-algebra

Definition

Action of L_∞-algebra L on manifold $\mathcal{M}$: L_∞-morph. $\mathsf{L} \to \mathfrak{X}(\mathcal{M})$.

Note: $\mathcal{V}_2(M)$ only encodes forms, not symmetric tensors.

Extension of Poisson bracket

$$\{-,-\}: \ \mathcal{C}^\infty(\mathcal{M}) \times T(\mathcal{M}) \to T(\mathcal{M})$$
$$\{f, g \otimes h\} := \{f, g\} \otimes h + (-1)^{(n-|f|)|g|} g \otimes \{f, h\}$$

Definition: Extended tensors

Let L be L_∞-structure on $\mathcal{M}$. Extended tensors are elts. of $T(\mathcal{M})$ such that elements of L act on it via $X \triangleright t := \{\delta X, t\}$.

Action Functionals

Action functionals are readily constructed from the generalized metric.

- Extended tangent bundle has structure group $GL(m, \mathbb{R})$
- Reduce to subgroup H by coboundary: $h_{ij} = \gamma_i h_{ij} \gamma_j$
- Factor out H-equivalent coboundaries: $\mathcal{H}_i = \gamma_i^* \gamma_i$.

Action functionals for generalized metric $\mathcal{H}$, dilaton d:

$$S = \int_M e^{-2d} \, d^D x \big(c_0 \mathcal{H}_{MN} \partial^M \mathcal{H}_{KL} \partial^N \mathcal{H}^{KL} + c_1 \mathcal{H}_{MN} \partial^M \mathcal{H}_{KL} \partial^L \mathcal{H}^{KN}$$
$$+ c_2 \mathcal{H}^{MN} (\mathcal{H}^{KL} \partial_M \mathcal{H}_{KL})(\mathcal{H}^{RS} \partial_N \mathcal{H}_{RS}) + c_3 \mathcal{H}^{MN} \mathcal{H}^{PQ} (\mathcal{H}^{RS} \partial_P \mathcal{H}_{RS})(\partial_M \mathcal{H}_{NQ})$$
$$+ c_4 \partial^M d \partial^N \mathcal{H}_{MN} + c_5 \mathcal{H}_{MN} \partial^M d \partial^N d \big)$$

Examples:

- No reduction, γ_i are vielbeins, $\mathcal{H} = (g_{\mu\nu})$, $\Rightarrow$ GR
- $GL(2D, \mathbb{R}) \to O(D, 1)$, yields GR + 1-form gauge potential

$$\mathcal{H}^{mn} = \begin{pmatrix} g^{\mu\nu} & -g^{\mu\nu} A_\nu \\ -A_\mu g^{\mu\nu} & 1 + g^{\mu\nu} A_\mu A_\nu \end{pmatrix}$$

- $GL(2D, \mathbb{R}) \to O(D, D)$ yields GR + 2-form gauge potential
  Generalized Metric of Generalized Geometry.

Comments on the Global Picture

The global picture seems to be in reach now.

- Surjective submersion $Y \twoheadrightarrow M$, $Y^{[2]} := Y \times_M Y$, $Y^{[n]} := ...$
- For example, $Y = \sqcup_i U_i$, $Y^{[2]} = \sqcup_{i,j} U_i \cap U_j$
- Gerbe is a principal $\mathsf{U}(1)$-bundle over $Y^{[2]}$ + data over $Y^{[3]}$
- Trivial gerbe: $Y = M$, $Y^{[2]} = M$, $\mathscr{G} = M \times \mathsf{U}(1) = M \times S^1$.
- T-duality on $M \times S^1$: trivial gerbe, $H = \mathrm{d}B$ globally.

Non-trivial gerbe in Generalized Geometry

- $\mathcal{V}_2(M) = T^*[2]T[1]M$ and $Q = \xi^\mu \frac{\partial}{\partial x^\mu} + p_\mu \frac{\partial}{\partial \zeta_\mu}$ only locally
- Assume nontrivial gerbe $H, B_{(i)}, A_{(ij)}, h_{(ijk)}$
- Hitchin's Generalized tangent bundle over patches i, j:
  $(X_{(i)}, \Lambda_{(i)}) = (X_{(j)}, \Lambda_{(j)} + \iota_X \mathrm{d}A_{(ij)})$
- $Q = \xi^\mu \frac{\partial}{\partial x^\mu} + p_\mu \frac{\partial}{\partial \zeta_\mu} - \frac{1}{3!}(\frac{\partial}{\partial x^\nu} H)\frac{\partial}{\partial p_\nu} + \frac{1}{2!}H_{\nu\mu_1\mu_2}\xi^{\mu_1}\xi^{\mu_2}\frac{\partial}{\partial \zeta_\nu}$

- L_∞-algebra of symmetries of DFT acts as Lie algebra
- Lie algebra can be integrated   Hohm, Zwiebach, 2012
- Proposal: Patch local descriptions by finite DFT symmetries
          Berman, Cederwall, Perry, 2014
- Papadopoulos 2014: This only works for trivial gerbes
- No surprise, DFT reduces to GenGeo, where we need to twist!
- Need to twist C-/D-bracket, just as in GenGeo.
- Twist can be defined and studied in our framework.
- We recover twists of Generalized Geometry as special cases.
- Integrate twisted action
- Global picture: Patch together with twisted transformations!

Summary:

✓ Full algebraic and geometric picture for local DFT

✓ Picture is extension from GR coupled to n-form fields

✓ weakened section condition from algebra

✓ twist of symmetry Lie 2-algebra

✓ Initial studies of global picture and Riemannian Geometry

Soon to come:

▷ Exceptional Field Theory (M-theory)

▷ Full Extended Riemannian Geometry

▷ Global Picture

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Summary and Outlook.

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